Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 254 NOVEMBER, 1952 No. 5

RECORDS OF RESEARCH *

BY

JOSEPH GRAY JACKSON [1]

Edwin F. Northrup[2] had above his desk this motto by Slosson:

> We may not find what we are looking for, but we never find anything unless we are looking for something.

Imbued with this attitude, the research worker finds vistas opening up at every turning which he never has time to investigate, and is constantly urged forward to the next experiment to find what is hiding around the ever-changing corner. In the light of this attitude, it is natural that he becomes very impatient over paper work, and is inclined to slight the keeping of proper research records.

In fact the temptation to ignore or slight the keeping of records at this point is very powerful. I remember from my own experience in experimental work—in going back I always wondered why my records were so skimpy; why I had not taken more complete data.

Since I have had occasion to look over numerous laboratory notebooks and other research records, I have noticed the same tendency—to get on horseback and rush forward to the next experiment, without thoroughly documenting what has been done. I consider that most research workers suffer from this tendency.

Why is it worthwhile to bear down on record keeping?

* On March 21, 1952, the Joint Committee of The Franklin Institute and the American Patent Law Association presented a program "Who is the First Inventor," in Franklin Hall, attended by an audience of 750. That meeting emphasized the importance of keeping proper research records. This article, prepared with the encouragement of Virgil Woodcock, President of the American Patent Law Association, is intended to discuss the requirements for research records.

[1] Member of the firm of William Steell Jackson and Sons, Philadelphia, Pa.; and Chairman of the Joint Committee of The Franklin Institute and the American Patent Law Association.

[2] A profound electrical experimenter, one of the founders of Leeds & Northrup, and later Vice President of Ajax Electrothermic Corporation.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

REASONS FOR KEEPING RESEARCH RECORDS

There are several valid reasons for keeping good research records:

1. *Avoid duplication.* Many times, research continuing over a period of years in a particular field returns at a later stage to subjects which were previously investigated. If on looking back at the earlier work, the records are fragmentary, obscure and confusing, the research group is strongly tempted to repeat the work, thus wasting time and money. Often the fact is that the original work was valid and well done, and the only defect is in the record keeping. A small additional expenditure of effort at the beginning would avoid much travel in circles.

2. *Take full advantage from negative results.* The temptation to slight research records is less pronounced in the case of very successful work than where the results are negative. Negative results are often very valuable as a guide to future research by the same organization or by others in the field, and they should be documented just as carefully as the occasional instance of great success.

3. *Development of fundamental approach.* Quite commonly at the beginning of a research program, workers are moving in the dark, and there is very little rhyme or reason to what is being done. This is especially true of the worker who is concerned with a small part of a coordinated program. The temptation is to take a very minimum of data on the features considered to be important, and not to make readings such as pressure or concentration, or not to make careful analyses of components, on the theory that such features are not important. Later, when a mass of data has been accumulated and it appears possible to develop a theory or fundamental approach, the earlier data are insufficient because they fail to include information on the controlling factors. More complete documentation of earlier work will often save a great deal of time in developing a fundamental approach to the problem based on earlier and later data.

4. *Publication.* Frequently at the time experimental work was undertaken, publication was not primarily in mind. The worker therefore has considered only the answering of his immediate question, and has ignored all other matters, or merely noted them on scraps of paper which did not get in the notebook and were not retained. Several times in the discussion of a technical paper I have heard an author asked a question as to some factor not reported. Too often I have heard him reply:

"Yes, we determined that, but I do not have the data."

5. *Research records for patent purposes.* Just as the experimenter often does not consider publication when he begins his work, he frequently is inclined to ignore patent aspects. Later if the subject matter becomes important, the experimenter turned inventor is unable to establish the facts necessary to prove that he is the first inventor of the subject matter. Too often this has led to the bitterly resented conclu-

sion that the Company which developed the subject matter is forced to take a license under someone else's patent on that subject matter.

SOME THINGS WHICH YOU MAY WISH TO ESTABLISH LATER

The very minimum of information which your research records should give should enable you to answer reliably the following questions based on your written records:

1. When the work was started.
2. Who did the work.
3. What was done, including when each step was taken.
4. What results were obtained.

CORROBORATION

The director of research and all other supervisory research personnel down to the group leader perform the important function of supervision. Some supervisor must know what the research worker is doing, why and how. Surely supervision is not performing its full function if the supervisor is not aware at the time what project is being worked on, what objective is sought and what method of attack is being used. Very shortly after any results are obtained, the supervisor should also know the data secured.

Likewise in the group actually at work on the project, there will usually be coworkers active on some related phase who must be kept informed of the work in question, in order that they can properly do their own work.

This is fortunate, as these people, the supervisor and the coworker, can very quickly and easily perform an additional function—that of corroboration from a legal standpoint. Thus with little additional effort, the research records can be kept in such a way as to satisfy better the main objective—that of improving the product or developing new products—and also of meeting the legal requirements for corroboration.

It must be remembered that if the researcher produces an invention by his research, testimony of the inventor and his written records[3] will be completely disregarded in determining who is the first inventor except to the extent that they are corroborated. This applies also to joint inventors—one joint inventor cannot corroborate another.

If your records, besides meeting the other objectives, are to be useful for patent purposes, they must provide for corroboration, and corroboration is so very important that it should be given primary emphasis. Inventors lose their rights as *first* inventors more because of lack of corroboration, than because of inadequacy of the content of the records themselves.

[3] The literature of this subject is rather limited. Kapp, *Laboratory Notebooks*, 33 JPOS 502 (1951) contains a good discussion on this special phase and cites new references. A very good practical discussion appears in Tuska, *Patent Notes for Engineers* (RCA 3rd ed. 1947) 80–89.

The first step in corroboration is the proper witnessing and dating of all original research records. It is very desirable to have an understanding witness sign and date each page of each notebook. The best form, frequently incorporated in a stamp to save all writing except the signature and date, is:

Witnessed and Understood
May 28, 1952.
John Jones (Signature)

Many people use the form:

Witness
May 28, 1952
John Jones (Signature)

and this is usually adequate. There is of course a possibility that the person who corroborated may not be available at the time that proof must be made, and it is therefore desirable to provide witnessing by two or more persons.

Since the inventor or inventors cannot corroborate, it may in some cases be convenient to set up "Witnessing teams" and provide periodically for witnessing of the notebook of A by X and Y who are perhaps supervisors or coworkers, but not joint inventors with A in the work.

Any person other than the inventor or coinventor will be of some value as a witness. In many cases, however, where the subject matter is complicated, a witness who is not specially familiar with the subject matter will not be of much assistance, and it is strongly recommended that the witness should have a complete understanding of the subject matter at the time he witnesses. The witness as a practical matter should know enough about the invention at the time he signs so that if the inventor died the information would not be lost.

It is of course best to use a witness who is as disinterested as possible. Members of the inventor's family are not desirable as witnesses. Though not essential, a witness who is not even a fellow employee will certainly be disinterested.

In a continuously running notebook, the location of the witnessing on the page is not very important, and the top or side margin or space at the bottom of the page is commonly used. Where the document is independent in form, as a report, the witnessing should at least be at the end and immediately below the end of the report, rather than at the bottom of the page where the report ends near the top. Even in the case of reports, witnessing on each page may be desirable where the subject matter is important.

It is of vital importance that the witness put down *only the date on which the witnessing takes place*, no matter how much later it may be than the date on which the work was done. Obviously witnessing of a notebook at frequent intervals, preferably at least once a week, is de-

sirable in order that the corroboration may be nearly simultaneous with the original work.

Witnessing has the effect of disqualifying the witness to claim originality of the subject matter. Thus in an amusing case, AB, the Director of Research, witnessed an inventor's disclosure. Later, in delivering a paper before the Blank Society, he claimed originality of the subject matter, and this claim was thoroughly discredited by circulating lithographs of the witnessed disclosure through the audience during the discussion of the paper.

CORROBORATION OF EXPERIMENTAL WORK

What has been said above applies to corroboration of documents. Corroboration is equally important on experimental work which may be urged to constitute a reduction to practice. In the absence of other proof, the first to reduce to practice, that is, first successfully to test a full sized operative embodiment of the invention, is considered to be the first inventor.

In the case of experimental work, the complete information will be contained in the notebook, and, if the development is important, will be embodied in a report. Corroboration as explained above is important in proving these documents, but it will not be sufficient by itself to prove the reduction to practice. It will be necessary to have corroboration by someone who witnessed the successful test and understood what was being done.

The important feature on which most corroborating witnesses of experimental data fall down is that the witness must be prepared to establish of his own knowledge that the subject matter which underwent the test corresponded to the invention. Thus if the invention involves an electric circuit, the witness must independently trace the circuit at the time of the test, and be prepared to establish that the circuit conformed to the one contained in the notebook or report. Likewise in the case of a chemical reaction, it is vitally important that the witness who corroborates shall verify by his own knowledge the reagents used.

The witness to be of greatest service for purposes of corroboration should make his own dated written record establishing what took place in the test.

The requirement that the test be successful is a very important one from the standpoint of the corroborating witness. Quite usually research workers have in mind commercial standards of acceptability, and they are very prone to "foul their own nest" by indicating that the subject matter was unsatisfactory, meaning that it did not measure up to commercial standards. Often, however, such statements are made when the test nevertheless completely demonstrated the satisfactory operation of the process from both the scientific and patent standpoint. It would be very desirable if all research workers reporting test results would keep in mind the important distinction between the success of

the test as demonstrating the operability of the subject matter, on the one hand, and the fact on the other hand that further improvement in speed of operation, precision, or the like would nevertheless be desirable to meet commercial demands. It is often considered by patent attorneys that a reduction to practice cannot be proved in the face of a slurring statement blunderingly made by someone witnessing the experiment, recorded in a contemporaneous document which was intended by the writer to mean that the device operated to perform the intended function, but that further improvement was required to meet commercial standards, but which is interpreted by the Court to indicate that the result was a failure.

HAS THE SUBJECT MATTER CHANGED?

When a person is asked to corroborate a document which he has witnessed at some time in the past, he will almost surely be asked whether the subject matter has changed since witnessing. This is often very difficult to answer if the document is one on which additions or modifications could readily be made. Quite frequently the honest witness can go no further than to say that the document "appears to be" in the form in which he witnessed it.

Obviously any form of presentation of the material which increases the difficulty of change, increases the likelihood that it can be established that the document is unchanged. Typewriting and ink are therefore preferable to pencil. Many persons expecting to face this situation produce a special document for witnessing which will be very difficult to change. Thus it is desirable to witness on a photostat or blueprint, since the inherent character of the photostat or blueprint tends to indicate that no change has occurred subsequent to the witnessing.

Notarizing of the document can be used to advantage as another assurance against the possibility of change, and as providing an impartial person to prove the existence of the document in that form. The notary's seal can be affixed in different pages, and can be impressed at important points in the document, if necessary by folding a page.

A popular misconception exists that the sending of a registered self-addressed envelope containing the disclosure to the inventor is a safe way of providing corroboration and evidencing absence of change. This is a popular fallacy, and this procedure cannot be recommended. The fact that an envelope which has remained in the possession of an inventor is sealed at the time of opening does not establish that it remained sealed. It would be very easy to open such a self-addressed envelope and fraudulently insert a disclosure.

FORMAL STEPS SIGNIFICANT IN RESEARCH

Certain milestones in connection with research should be formalized in order to enable the company to determine the progress being made,

and the status at any stage. These same formal steps are of great assistance in tying together various records for patent purposes.

The practice of many companies of assigning a project number when research is first undertaken is applauded by all accounting and tax personnel and widely recommended. The project number should be formally taken out under a specified date in a book under the control of a reliable person other than the inventor. The formal record of issuance of the project number should clearly set forth the subject matter to be included under that project number, and when the subject matter changes, an appropriate amendment should be made under a later date in the project number book, or a new project number should be issued.

In many organizations, a research or development or patent committee considers research progress, often indicating the relative success of the work to date and the action to be taken in the future. Formal, dated, written minutes of the action of any such committee should be kept, and the subject matter acted upon should be clearly identified by project number or otherwise. The actions of such a committee will in many cases be significant from the standpoints of conception, diligence or reduction to practice.

When a particular item has been developed to the extent that it is to become a standard item of production, formal action should be taken to standardize the item, and it should be assigned a number different from the project number. A standardizing number should be taken out under a recorded date in a book kept for the purpose under the jurisdiction of a reliable person other than the inventor. The standardizing book should show what the item is which is standardized and refer to the project number and to the drawings, if any, of the item. Aside from the use in showing completion of the research stage, the standardizing of the item may be very important not only as establishing reduction to practice, but also as showing when the subject matter went into public use, and possibly anticipated patents granted to others.

All records by research workers, development shop, purchasing, and the like, should refer to the proper project number or the standardized item number for identification.

ORIGINAL CONTEMPORANEOUS RECORDS OF EXPERIMENTATION

There are great advantages in making the records of experimentation immediately at the time the data are taken, in a notebook for the purpose. Many workers fear criticism for making a messy illogical record, and so make original notations on scratch notes which are transferred to a notebook. This is a serious mistake, as it involves possibility of error or incompleteness in transfer of data, and also places the original entry, which may be highly important for proof, in a nonpermanent form.

Some research people also err by making erasures in the notebook, thus throwing doubt not only on the subject matter which is erased but

also on all other entries. It is much better to strike out errors and make corrections.

Obviously the original notebook is best kept in ink, although pencil notations are usually sufficient.

The use of separate data sheets for special experiments is convenient, but it is believed that the advantage from the standpoint of permanence in having everything in the notebook outweighs the advantages of the separate data sheets. Where separate data sheets are used, they should, of course, be witnessed like the notebook.

The question of whether the notebook is bound or looseleaf is of less importance if corroboration is provided by witnessing on each page.

Many workers use a card system to record data. Carlton Ellis made extensive use of this system and the cards were carefully witnessed for corroboration.

Sound records have many features to commend them as original records of research. The experimenter in reading an instrument can very conveniently dictate the readings, along with the date and the basic lay-out of the experiment, to a sound recorder and persons corroborating can at intervals readily dictate on the same record the date and the information regarding the corroboration. It is believed that more extensive use of the sound record will be made right at the point at which experiments are conducted. To complete the chain of proof, the typed transcription should, however, be separately witnessed as a document.

STEP BY STEP RECORDS IN ORDER OF TIME

The laboratory notebook usually gives a very poor day-by-day record of the activities of a research worker, because items which do not relate directly to experimental work which obtains data to be recorded, usually do not appear in the notebook. Thus such things as library search, examination of equipment, making equipment, and the like are not included. It is very desirable to be able to establish all the activities of a research worker from day to day, in reference to what is done, and what project it relates to. This is important as a time record and also from the standpoint of proving diligence.

It is best to keep a diary or log showing what is done throughout the day and on what project the work is to be charged. It is similarly important in many cases to prove the activities of mechanics and production workers insofar as they relate to development projects. A study should be made of the time-record-keeping procedure of development shops and the like, and this should be tied in with the research projects where it is devoted to research and development. I have recently had occasion to deplore the practice of some plants of throwing away time records of the development shop after a few years. These can prove very valuable even after 20 years.

DRAWINGS

From the standpoint of development, the most important feature to establish about a drawing is its date. The numbering system used on drawings is therefore important. The practice of many companies of making a record of taking out the drawing number is commendable. The record should be under the control of someone other than the draftsman, and should show the subject matter to which the drawing number is applied. It is usually bad practice to issue drawing numbers out of order or to assign blocks of numbers to a particular item at the beginning, as this defeats the possibility of proving the date of the drawing by the date of issuance of the drawing number. Drawing dates are important on contract, tort liability, taxation and patent matters.

The practice should be clearly established as to whether the drawing is to be dated when it is started or when it is completed.

The drawing numbering system should make it possible to establish reliably the draftsman's identity. In many cases this is included on the drawing, and this may be a great convenience.

It is furthermore important to establish the date of changes in the drawing, as frequently the critical feature in a contract or an invention revolves around the date of change in a drawing. The practice of changing drawings without indicating revisions on the drawing and without recording and indicating dates of revision is to be regretted. When the drawing revision is made there should be a master record other than the drawing itself which shows the date of completion of a particular revision.

The coding of drawing dates is usually undesirable. I recently encountered a case in which the coding was known only to one man in the organization, and depended entirely on his recollection to interpret.

It is often important to establish that certain environmental features were used with the drawing showing the critical aspect of the invention. These environmental features are often present either in an assembly drawing or in standard parts drawings. Adequate cross referencing to the assembly or standard parts drawings on the drawing of the new subject matter therefore becomes important. This is sometimes done by parts lists and bills of materials, which should be subject to the same formalities as drawings.

SPECIFICATIONS

A very good practice which is used in some large organizations such as the Ordnance Department is to issue a specification on an item under development before it is standardized. The specification thus becomes a valuable record of research. It encourages the research worker to spell out in complete detail the subject matter under development, and thus avoids the difficulty commonly encountered that the research worker assumes too much knowledge on behalf of the reader and gives too little

information in his notebook and his reports as to the environment in which the invention is used.

PHOTOGRAPHS

It is important, particularly from the standpoint of reduction to practice, to establish the identity of the subject matter which was in existence and was tested at a particular time. Photographs lend themselves readily to this purpose and should be liberally used as research records. Usually it is desirable to go on a photographic splurge, and take photographs from all angles, in different positions of operation, interiors and exteriors, and exploded line-ups of disassembled parts.

In the case of a test, the photograph will desirably provide extraneous evidence of corroboration, and should show the various people present, and if possible in the process of making the test.

Dating should always be recorded in the photograph itself to guard against confusion in records by the photographer. A sign can desirably be included in the photograph showing the date, or a newspaper of that day with the date exposed can be included in the picture.

SAMPLES

All those concerned with direction of research have had occasion to regret the failure to identify, and the tendency to destroy samples. Samples should always be clearly marked and identified as to date and subject matter. Even at the expense of building special sample storage facilities, samples should be carefully retained.

The sample identification should give as much pertinent information as possible, and particularly should indicate by what process the sample was produced, by whom, and when and under what special circumstances.

There is a great tendency in research using assemblages of standard equipment to cannibalize a device which has successfully performed in one test in order to make tests on other subject matter. Of course it is realized that major items of equipment such as generators, cathode ray oscillographs, meters, and the like, are not available in indefinite numbers, and must be used in different tests successively. Wherever possible, however, the features of the successfully tested equipment which are peculiar to that device should be retained intact.

SECONDARY RECORDS OF EXPERIMENTATION

Primary records of experimental work, such as laboratory notebooks, are of first-rate importance because they are the earliest in existence, usually the most complete, and they carry inherent evidences of authenticity in relation to the circumstances. Every research director should, however, be alert to create adequate secondary records of experimentation.

Progress reports often prove to be valuable sources of information as to what was done within different periods. A master set should be

available in unbroken sequence, and those reports dealing with a particular project should be collected in a file relating to that project.

On completion of the project, a report which spells out what was done in considerable detail should be made. This report at completion of the project should preferably not be merely a sketchy analysis of the experimental work with amplification of the conclusions, but should have the experimental work outlined in considerable detail. It should be realized that often the notebook will be difficult or impossible to interpret after the research worker is no longer available, and this report should stand on its own feet to explain exactly what was done.

What has been said above about the numbering of drawings and about witnessing of notebooks applies also to the numbering and witnessing of secondary research reports.

In an elaborate research program, the finding of pertinent reports is often extremely difficult, and the circulation should be ample to assure that each group concerned will have complete files. I have recently had occasion to look up the work done by a moderate sized company on a particular project, and after collecting all the data apparently available, have found that other departments not supposed to be concerned with the matter have files with a great deal of additional data which no one previously located.

Careless and loose comments made on buck slips and the like in regard to research reports should be avoided, as they may result in discrediting work which really was successful in indicating the operability of the invention from scientific and patent standpoints.

Often valuable supporting evidence of research will be found in reports of service organizations, such as chemical and metallurgical laboratory reports, and these should be properly keyed and copies should be retained in the main project file.

CODES AND ABBREVIATIONS

In interpreting research records long after the event, codes and abbreviations become of great importance. Difficulty is frequently encountered through the tendency of the drawing room to specify materials in abbreviations which are later not understood. A suitable set of drawing standards such as the Ordnance drawing standards should be adopted by the drawing room and all codes and abbreviations should conform to these standards. Any special abbreviations should be footnoted on the drawings.

The reader should not be left to guess what the nature of the subject matter is to which the drawing relates, and this should either be set forth in considerable detail on one drawing of the set, or should be set out by cross referencing.

The problem of codes is extremely difficult in industrial chemistry. Many chemical plants manufacturing specialties are beset with the fear that employees will leave knowing their formulae and take away im-

portant customers. Chemical plants commonly use codes for reagents, and often the reagent is purchased under the code and arrives from the supplier merely identified by the code number. This practice is overdone, and in many cases greater efficiency would exist if the producer disregarded the coding altogether and described the ingredient by its correct name.

In proving records of research, great difficulty is caused by the tendency to use the code names and numbers even in the basic research.

I remember visiting a laboratory and inquiring about some simple reagents to perform an experiment. I was told that, "Sulfuric acid is in that bottle marked 'ammonia' and nitric acid is in that other bottle marked 'ferrous sulfate.'" This practice went through the various chemicals and the chemist relied on memory to keep him straight. I remember the confusion caused in proving a plastic formulation by a paint company due to the fact that they periodically changed their coding. They testified that "Q336" meant manganese dioxide up to July 5, 1936, and after July 5, 1936 it designated methyl alcohol.

When it is realized that in order to corroborate a test the witness must know of his own knowledge what ingredients were employed, it will be appreciated that coding in this form imposes tremendous obstacles in the way of proof.

I recently observed a witness testify about metallurgical experiments in which he had participated ten years ago. The data sheets had many abbreviations. Some were never explained and others were merely "guessed" about. Yet they had seemed obvious at the time.

PURCHASING, PRODUCTION, AND SHOP RECORDS

Very often corroboration of a certain development can be obtained by establishing the purchase for that development of an item which was not previously used in the plant. Even if the item was one normally used, the purchase on account of that project of the materials for it may help to corroborate. Purchasing records, if properly retained and properly indicating the project, may be of great value as records of research. Similarly, shop orders and shop time records applied on research and development are significant research records, and the research department should control the information given on such records and the length of time that they are retained.

Likewise production records are important as research records where they show the change to produce a new item and where they show testing or inspection of a new item. Under such circumstances the production department should maintain the same records as would be indicated if the work was done in the laboratory.

SPECIAL RECORDS FOR PATENT PURPOSES

In all of the discussion above, we have been concerned with records which should properly be made in connection with research and development for purposes other than patent purposes, and have indicated the

features in which these records should be strengthened to be of maximum benefit from the patent standpoint.

There are, however, certain records which are peculiar to the patent aspect and should now be considered.

When subject matter is developed and appears to have patent possibilities, a record of conception[4] should be made. Some companies delight in producing record of conception forms with blanks for everything imaginable that might ever happen in connection with the conception of an invention, and to my mind they thereby create hazards for the inventor to overcome.

Every so often a research worker follows the form which he finds in a patent and writes out a record of conception in the form of a complete patent specification, probably thinking that he is going to incur the undying gratitude of the patent attorney. Unfamiliar with the purpose of the various features in a patent specification, the inventor usually merely confuses the issue and makes the patent attorney's job much more difficult in searching for crumbs of disclosure in an ocean of attempted broadening statements.

To my mind the proper approach is a simple, down-to-earth statement of the subject matter with explanation of any obscure terms or ideas. A very satisfactory formula is this: (1) show what was wrong in the prior art; (2) show how this is corrected by the present invention; (3) describe the details of the present invention and tell how it works in your own way; (4) describe any additional advantages and disadvantages; (5) list what is believed to be new.

The record of conception should, of course, be accompanied by drawings, which will usually be free-hand sketches, and the description should describe the drawings, preferably with reference characters if the drawings are complicated.

Corroboration of the record of conception is very important, and what is said above about witnessing and notarizing applies.

The covering letter which transmits the record of conception to the patent department or patent attorney offers a splendid opportunity to record what has happened in connection with the development up to that time. This covering letter should preferably include a statement as to whether the device has been made and whether it has been tested, and to what extent it has been commercially used or sold, with dates.

The other records, such as records of witnessing of tests, records regarding drawings, and the like, which are normally used for patent purposes in connection with research will follow the forms indicated above, and need not be especially constructed for patent requirements. Of course the patent attorney and the patent department will maintain records of their own which do not come within the scope of the present discussion.

[4] Various other names are used, such as "patent disclosure," "patent form," "record of priority," or "record of invention."

PUBLICATIONS AS RESEARCH RECORDS

Publications perform certain functions as research records, and they have special significance in patent matters.

Publication in a United States patent after one year will bar patenting of the subject matter in the United States on a later filed application and therefore protect your Company against the possibility of being stopped in the production of the item due to the granting of a later patent. It may, therefore, be wise to include in the specification of a patent, disclosure which is much more complete than would be required as foundation for the claims, in order to establish a public record of the subject matter.

In some industries where the know-how has been largely concealed in the using plants, the publication of text books describing current practice in detail is a recognized technique to prevent improper issuance of patents on items which are old.

The issuance of technical articles describing your research necessitates that any patent applications in the United States on the subject matter be filed within one year and bars the filing of patent applications in most foreign countries. Some concerns have adopted the technique of deliberately making publications of research activity in obscure publications or obtaining patents in obscure foreign countries to avoid difficulty by patenting of the subject matter by others at later dates.

CONCLUSION

The research director and the group leaders responsible for the maintenance of research records will do well to review from time to time the technique being followed in maintaining the records under their jurisdiction. A legitimate procedure is to spot check the records on particular projects from time to time, and taking a dispassionate view, endeavor to decide how well the records meet the requirements. Just because you have the matter spelled out in a "standard procedure" or "company regulation" does not help if in the individual case which comes up you find that the records are deficient.

Originality knows no definite rules or limits, it certainly is not restricted to college graduates, and when you find this rare quality in a research worker, you naturally wish to promote it. Often a person with originality has eccentricities, and if he is a research worker, the chances are ten-to-one that they include a desire to avoid paper work. In extreme cases the only alternative is to provide a coworker having secretarial functions to work along on the project and keep the necessary records.

The important thing is to get full value for the research dollar not only by getting the results, but also by keeping the records which will give security to the results.

APPLICATION OF MACLAURIN SERIES TO THE ANALYSIS OF BEAMS IN BENDING

BY

M. HETÉNYI[1]

1. INTRODUCTION

The elementary flexure theory of beams is based on the assumption that plane cross-sections of a beam remain plane after bending deformation has taken place. In considering part of such a beam subjected to M bending moment (Fig. 1*a*), we find that the strain ϵ in a fiber at z

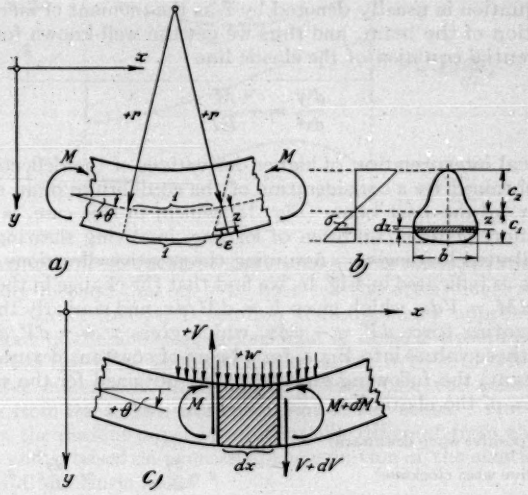

FIG. 1.

distance from the neutral line, or line of zero extension, can be expressed by means of the radius of curvature r, as $\epsilon = z/r$. If we regard as positive curvature the one that is concave when viewed from above, and as positive slope the one obtained by clockwise rotation, for small values of the slope we may write $\theta \approx dy/dx$, where $y = f(x)$ is the elastic line of the deflected beam. Consequently we get $1/r = -d\theta/dx = -d^2y/dx^2$. Substituting this latter value of $1/r$ into the above expression for

[1] The Technological Institute, Northwestern University, Evanston, Ill.

ϵ, and introducing the modulus of elasticity E of the material, according to Hooke's law the bending stress σ at any z distance from the neutral line is obtained as

$$\sigma = \epsilon E = \frac{z}{r} E = - zE \frac{d^2y}{dx^2}. \qquad (a)$$

Thus we find that the assumption of a plane cross-section leads to a linear distribution of the bending stresses. Since these stresses will have to balance the external bending moment M acting on the beam, equilibrium requires that

$$M = \int_{-c_2}^{c_1} b\, dz \cdot z = - \frac{d^2y}{dx^2} E \int_{-c_2}^{c_1} z^2 b dz, \qquad (b)$$

the notation being that shown in Fig. 1*b*. The integral on the right side of this equation is usually denoted by I as the moment of inertia of the cross-section of the beam, and thus we get the well-known formula for the differential equation of the elastic line

$$\frac{d^2y}{dx^2} = - \frac{M}{EI}. \qquad (c)$$

Physical interpretation of higher derivatives of the deflection curve may be obtained by a consideration of the equilibrium of an elemental length, dx, of the bent beam (Fig. 1*c*) which, in this case, is assumed under a more general condition of loading, involving shearing force V and distributed loading w. Assuming the positive directions for these quantities as indicated in Fig. 1*c*, we find that the change in the bending moment $dM = Vdx$, which gives $V = dM/dx$, and similarly the change in the shearing force $dV = - wdx$, which gives $w = - dV/dx$. Substituting these values into Eq. *c*, for a beam of constant flexural rigidity (EI constant) the following expressions are obtained for the successive derivatives of the elastic line

Deflection: (positive when downward)	$y = f(x)$			(1*a*)
Slope: (positive when clockwise)	$\theta = \frac{dy}{dx}$;	or	$\frac{dy}{dx} = \theta$	(1*b*)
Bending moment: (positive when clockwise on left side)	$M = - EI \frac{d^2y}{dx^2}$;	or	$\frac{d^2y}{dx^2} = - \frac{M}{EI}$	(1*c*)
Shearing force: (positive when upward on left side)	$V = - EI \frac{d^3y}{dx^3}$;	or	$\frac{d^3y}{dx^3} = - \frac{V}{EI}$	
Distributed loading: (positive when downward)	$w = EI \frac{d^4y}{dx^4}$;	or	$\frac{d^4y}{dx^4} = \frac{w}{EI}$	(1*d*)
Rate of change of distributed loading: (positive when increasing)	$\frac{dw}{dx} = EI \frac{d^5y}{dx^5}$;	or	$\frac{d^5y}{dx^5} = \frac{1}{EI} \frac{dw}{dx}$	(1*e*)
Rate of change of dw/dx: (positive when increasing)	$\frac{d^2w}{dx^2} = EI \frac{d^6y}{dx^6}$;	or	$\frac{d^6y}{dx^6} = \frac{1}{EI} \frac{d^2w}{dx^2}$	(1*f*)

This sequence may be continued indefinitely by taking higher derivatives of the distributed loading diagram, in case such higher derivatives do not vanish.

The above set of equations, shown also graphically in Fig. 2, constitutes the "biography" of the bent beam. It contains all the char-

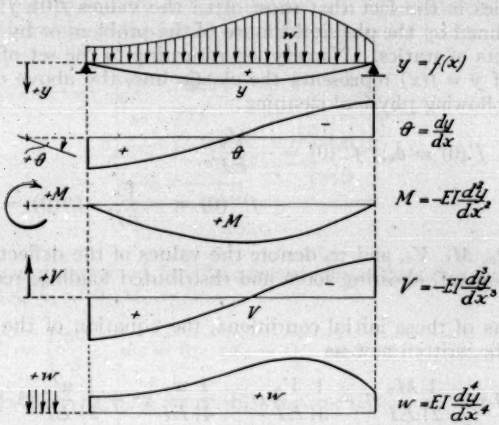

FIG. 2.

acteristic features of the problem and consequently furnishes the foundation for any method that may be used for the determination of the elastic line. In an analytical solution the customary approach is to start with one of the above equations that is defined by conditions of statics, such as Eq. 1*c* for instance, and to derive from it the elastic line by successive integration, determining in each case the constants of integration from the known end-conditions of the beam. The method proposed in the present paper is substantially different from this usual procedure, and is based on expressing the equation of the elastic line in the form of a Maclaurin series.

2. REPRESENTATION OF THE ELASTIC LINE BY MACLAURIN SERIES

It is known from elementary calculus that the value of a continuous function $y = f(x)$ may be determined at any x distance from the origin (where $x = 0$) by a Maclaurin series as

$$y = f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n + \cdots, \qquad (2)$$

where the symbols $f(0), f'(0), f''(0)$, etc. denote the values of the func-

tion $f(x)$ and its successive derivatives at $x = 0$, that is

$$f(0) = (y)_{x=0}, \quad f'(0) = \left(\frac{dy}{dx}\right)_{x=0} \quad \text{and} \quad f''(0) = \left(\frac{d^2y}{dx^2}\right)_{x=0}.$$

The great advantage obtained by using Eq. 2 in beam deflection problems lies in the fact that most often the values $f(0)$, $f'(0)$, $f''(0)$, etc. are defined by the physical nature of the problem or by the simple requirements of statics. Namely, on returning to the set of Eq. 1, we find that if $y = f(x)$ represents the elastic line, the above derivatives have the following physical meaning:

$$f(0) = y_0; \quad f'(0) = \theta_0; \quad f''(0) = -\frac{M_0}{EI};$$

$$f'''(0) = -\frac{V_0}{EI}, \quad f^{iv}(0) = \frac{w_0}{EI}, \quad \text{etc.},$$

where y_0, θ_0, M_0, V_0, and w_0 denote the values of the deflection, slope, bending moment, shearing force and distributed loading, respectively, at $x = 0$.

In terms of these initial conditions, the equation of the deflection curve can be written now as

$$y = y_0 + \theta_0 x - \frac{1}{2!}\frac{M_0}{EI}x^2 - \frac{1}{3!}\frac{V_0}{EI}x^3 + \frac{1}{4!}\frac{w_0}{EI}x^4 + \frac{1}{5!}\frac{w'_0}{EI}x^5 + \cdots. \quad (3)$$

As an illustration let us consider the cantilever beam in Fig. 3, in

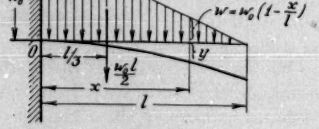

FIG. 3.

which case we know in advance that $y_0 = 0$, $\theta_0 = 0$,

$$M_0 = -\frac{l}{3}\frac{w_0 l}{2}; \quad V_0 = \frac{w_0 l}{2}; \quad w_0 = w_0; \quad w_0' = -\frac{w_0}{l}.$$

Since all higher derivatives of w identically vanish, the infinite series in Eq. 3 is reduced in this case to a polynominal of the 5th degree, and the equation of the elastic line can be written directly as

$$y = \frac{1}{2}\frac{w_0 l}{6EI}x^3 - \frac{1}{6}\frac{w_0 l}{2EI}x^3 + \frac{1}{24}\frac{w_0}{EI}x^4 - \frac{1}{120}\frac{w_0}{lEI}x^5,$$

or, in a form that is more convenient for calculation:

$$y = \frac{w_0 l^4}{120EI}\left(10\frac{x^2}{l^2} - 10\frac{x^3}{l^3} + 5\frac{x^4}{l^4} - \frac{x^5}{l^5}\right).$$

The method, which may be suitably named as the method of initial conditions, can be equally well applied to such a case like the one shown in Fig. 4, where one of the initial conditions, θ_0, is not known in advance.

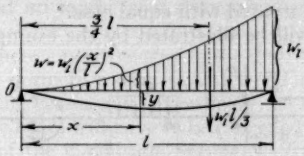

FIG. 4.

Leaving θ_0 undetermined for the present, since the other initial values are known as $y_0 = 0$; $M_0 = 0$;

$$V_0 = \frac{1}{4}\frac{w_1 l}{3}; \quad w_0 = 0; \quad w'_0 = 0; \quad \text{and} \quad w''_0 = 2w_1/l^2,$$

the rest of the derivatives vanishing, we can write

$$y = \theta_0 x - \frac{1}{6}\frac{w_1 l}{12EI}x^3 + \frac{1}{720}\frac{2w_1}{l^2}x^6.$$

However, the prescribed end-condition $(y)_{x=l} = 0$ has not yet been introduced in the calculation. By means of the above formula this can be stated as

$$(y)_{x=l} = \theta_0 l - \frac{w_1 l^4}{72EI} + \frac{w_1 l^4}{360EI} = 0,$$

which defines $\theta_0 = w_1 l^3/90EI$. Substituting this value for θ_0 in the above expression for y, after suitable grouping of the terms we have the final formula for the deflection curve

$$y = \frac{w_1 l^4}{360EI}\left(4\frac{x}{l} - 5\frac{x^3}{l^3} + \frac{x^6}{l^6}\right).$$

3. DISCONTINUITIES IN THE DERIVATIVES OF THE ELASTIC LINE

The use of a Maclaurin series for the representation of the elastic line of a bent beam seemingly depends on the condition that within the entire span the deflection curve and all its derivatives remain continuous functions, which was the case in the examples discussed thus far. It is known, however, that at the point of application of a concentrated force

there will be a step or discontinuity in the third derivative of the deflection curve. Similarly, a concentrated bending moment will cause a discontinuity in the second derivative, and a partially distributed loading will produce discontinuities in the fourth derivative at both ends of the loaded region. The method of initial conditions would not be of much value if these problems of greatest practical importance would have to be left out of consideration. It will be shown that this is not the case, and that by further application of the general solution of Eq. 3 all such discontinuities may be treated with equal ease.[2]

The procedure will be illustrated by the example shown in Fig. 5,

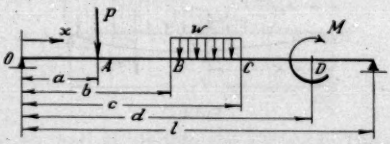

FIG. 5.

where there are four discontinuities in the derivatives of the elastic line, at points A, B, C, and D, respectively.

Taking the origin of the x coordinates at the left end of the beam, it being simply supported we know that $y_0 = 0$ and $M_0 = 0$. In terms of the other two, non-vanishing, initial values θ_0 and V_0 (from which only θ_0 should be regarded as unknown, since V_0 can be calculated from statics in any case), the equation of the deflection curve between points 0 and A can be written as $y_x = \theta_0 x - (1/6)\,(V_0/EI)x^3$. This form of expression, however, will not be correct beyond point A, where we know that a discontinuity of magnitude P occurs in the third derivative of the elastic line, which is proportional to the shearing force in the beam. One method of approach would be to calculate from the above formula, which is good up to $x = a$, the values y, θ, M and V at point A, and then on shifting the origin to this point we could start over again, with a repeated application of the same basic equation, but using the new initial conditions.

There is, however, a much more convenient way of dealing with the problem. Namely, we can take the point of view that the initial values θ_0 and V_0 at $x = 0$ will continue their influence beyond point A, only that their effect will have to be supplemented from point A onward by the action of force P, which is equivalent to a negative step of the same

[2] The basic idea of the method of initial conditions was first described by the author in 1937, in the appendix of a paper in the *Journal of Applied Mechanics*, entitled "Deflection of Beams of Varying Cross Section." There, however, only such cases were discussed where the elastic line was continuous in all its derivatives along the entire length of the beams. The method of approach to be used in case of discontinuities was discovered several years later, and is published here for the first time, although it has been regularly taught by the author at the Technological Institute of Northwestern University since 1946.

magnitude in the shearing force diagram of the beam. The effect of a shearing force on the deflection curve, on the other hand, has already been established in Eq. 3 by the expression $-(1/6)\ (V_0/EI)x^3$, where x is the distance measured from the point of application of the shearing force. In accordance with this, the change in the shearing force at $x = a$ being $-P$, its effect on the deflection line for all values $x \geqq a$ can be expressed as $(1/6)\ (P/EI)(x - a)^3$. Adding this corrective term to the previous expression, the equation for the deflection curve beyond point A, and up to the next discontinuity, can be written as $y_x = \theta_0 x - (1/6)(V_0/EI)x^3 + (1/6)(P/EI)(x - a)^3$. This formula is good up to $x = b$, where a positive step in the distributed loading w will contribute a further corrective term $(1/24)(w/EI)(x - b)^4$. At $x = c$ the distributed loading ceases and, accordingly, its continuation for the remaining part of the beam will have to be cancelled by the introduction of a negative term of the magnitude $-(1/24)(w/EI)(x - c)^4$. Finally at $x = d$, where a concentrated moment M is applied, the equation of the elastic line will have to be again adjusted by the addition of $-(1/2)\ (M/EI)(x - d)^2$ in accordance with Eq. 3, leading to an expression that is valid for the last portion of the span. The equation of the deflection curve for the five, individually continuous, sections of the beam in Fig. 5 is thus obtained in the following form:

$$\text{for } 0 \leqq x \leqq a, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3$$

$$a \leqq x \leqq b, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{6}\frac{P}{EI}(x-a)^3$$

$$b \leqq x \leqq c, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{6}\frac{P}{EI}(x-a)^3 + \frac{1}{24}\frac{w}{EI}(x-b)^4$$

$$c \leqq x \leqq d, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{6}\frac{P}{EI}(x-a)^3 + \frac{1}{24}\frac{w}{EI}(x-b)^4 - \frac{1}{24}\frac{w}{EI}(x-c)^4$$

$$d \leqq x \leqq l, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{6}\frac{P}{EI}(x-a)^3 + \frac{1}{24}\frac{w}{EI}(x-b)^4 - \frac{1}{24}\frac{w}{EI}(x-c)^4 - \frac{1}{2}\frac{M}{EI}(x-d)^2.$$

Putting $y_{x=l} = 0$ in the last of these equations, θ_0 can be determined and, this being the only unknown in the above formulas, the solution of the entire problem is thereby completed.

The method is obviously capable of further generalization, including discontinuities in y, θ, or in the derivatives of the distributed loading. Problems of this kind will not be discussed here, with the exception of one example, shown in Fig. 6, which is to elucidate the manner of approach in case of a distributed loading of variable intensity.

The simply supported beam in Fig. 6 is subjected to a linearly distributed loading between points A and B. The initial conditions for the left end of the beam are now $y_0 = 0$; $\theta_0 \neq 0$; $M_0 = 0$ and $V_0 = (1/2)w_m c(d/l)$, where $d = l - a - 2c/3$. Thus we have

$$\text{for } 0 \leqq x \leqq a, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3.$$

At $x = a$ we have $w = 0$. The rate of change of the distributed loading does not vanish, however, but equals w_m/c. Consequently, beyond point B the deflection curve can be written as

$$\text{for } a \leqq x \leqq b, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{120}\frac{w_m}{cEI}(x-a)^5.$$

At $x = b$ we must introduce corrections of two types, to discontinue both the magnitude w_m and the rate of change w_m/c of the loading at $x = c$. This is accomplished by the last two terms of the following equation.

$$\text{for } b \leqq x \leqq l, \quad y_x = \theta_0 x - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{120}\frac{w_m}{cEI}(x-a)^5 - \frac{1}{24}\frac{w_m}{EI}(x-b)^4 - \frac{1}{120}\frac{w_m}{cEI}(x-b)^5.$$

The significance of the last two terms is the same as if we would have added at $x = b$ a linearly increasing negative distributed load, shown by dotted line below the beam in Fig. 6, to cancel the continuation of the

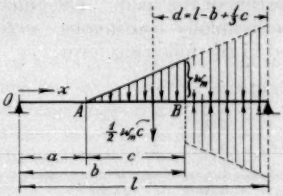

FIG. 6.

load on the beam beyond point B. Substituting $y_{x=l} = 0$ into the last equation, to satisfy the condition at the right end of the beam, we find for the initial slope

$$\theta_0 = \frac{w_m}{120EI}\frac{1}{cl}\{10c^2dl^2 - (l-a)^5 + 5c(l-b)^4 + (l-b)^5\}.$$

Introducing this value for θ_0 into the above three equations, the deflection curve for the three sections of the span is completely defined.

The method, in addition to being the fastest for problems of this kind, has two particularly attractive features: (1) the corrective terms entered for each new portion of the beam render the resulting formulas very clear and any errors easily traceable, and (2) the type of terms used, as $(x-a)$ and $(x-b)$, are most convenient when calculating deflection ordinates at these salient points.

4. STATICALLY INDETERMINATE PROBLEMS

The versatility of the method of initial conditions is perhaps best shown by the fact that it can be employed with the same ease for the

solution of statically indeterminate problems as for determinate problems. As a matter of fact, this distinction hardly needs to be made in working out the solution. For example, in the statically indeterminate case shown in Fig. 7*a*, out of the four conditions for each end of the beam two are prescribed (y and θ), and two are unknown (M and V). If the same beam were simply supported at its ends, which would make it statically determinate, y and M would then be defined in advance, and θ and V would need to be determined. The fact that in this latter case V could be calculated from conditions of statics does not make an essential difference if we consider the whole system of equations which can be derived from Eq. 3 by successive differentiation, namely,

$$y_x = y_0 + \theta_0 x - \frac{1}{2}\frac{M_0}{EI}x^2 - \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{24}\frac{w_0}{EI}x^4 + \frac{1}{120}\frac{w'_0}{EI}x^5 + \frac{1}{720}\frac{w''_0}{EI}x^6 + \cdots \quad (4a)$$

$$\theta_x = \theta_0 - \frac{M_0}{EI}x - \frac{1}{2}\frac{V_0}{EI}x^2 + \frac{1}{6}\frac{w_0}{EI}x^3 + \frac{1}{24}\frac{w'_0}{EI}x^4 + \frac{1}{120}\frac{w''_0}{EI}x^5 + \cdots \quad (4b)$$

$$M_x = M_0 + V_0 x - \frac{1}{2}w_0 x^2 - \frac{1}{6}w'_0 x^3 - \frac{1}{24}w''_0 x^4 - \cdots \quad (4c)$$

$$V_x = V_0 - w_0 x - \frac{1}{2}w'_0 x^2 - \frac{1}{6}w''_0 x^3 - \cdots \quad (4d)$$

$$w_x = w_0 + w'_0 x + \frac{1}{2}w''_0 x^2 + \cdots, \text{ etc.} \quad (4e)$$

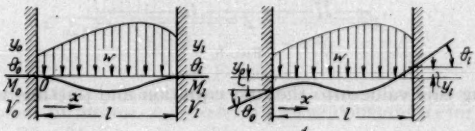

FIG. 7.

The third and fourth of the above equations are recognized as representing requirements of statics, namely, the equilibrium of moments and forces. Similarly the first and second equations can be regarded as expressing equilibrium, or continuity, requirements for deflection and slope. Which pair of these equations one should use in the solution will depend on what quantities are given in advance and which need to be calculated. If the conditions at the ends involve non-vanishing, but assigned, values for y and θ, such as shown in Fig. 7*b*, this will make no difference in the process.

As an example, let us consider a beam on three supports subjected to a linearly distributed loading as shown in Fig. 8, and pose the question as to what should be the position (l_1) and the elevation (y_B) of the intermediate support with respect to the two others, to make the three reactions equal. The requirement can thus be stated as $A = B = C = (w_0 + w_l)l/6$.

On the basis of Eqs. 4*a* and 4*c* we can write in this case, for $l_1 \leqq x \leqq l$,

$$y_x = \theta_0 x = \frac{1}{6}\frac{V_0}{EI}x^3 + \frac{1}{24}\frac{w_0}{EI}x^4 + \frac{1}{120}\frac{w'_0}{EI}x^5 - \frac{1}{6}\frac{B}{EI}(x - l_1)^3$$

and

$$M_x = V_0 x - \frac{1}{2}w_0 x^2 - \frac{1}{6}w'_0 x^3 + B(x - l_1),$$

where

$$V_0 = \frac{1}{6}(w_0 + w_1)l = B, \quad \text{and} \quad w_0' = -\frac{w_0 - w_1}{l}.$$

The unknowns in the right side of the above equations are θ_0 and l_1. The latter can be directly determined from the second equation by setting $M_{x=l} = 0$, which gives

$$l_1 = l\frac{w_1}{w_0 + w_1}.$$

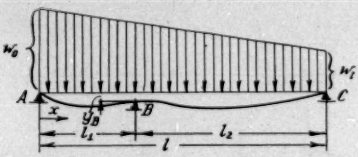

FIG. 8.

Substituting this value into the first equation and putting $y_{x=l} = 0$ we have

$$\theta_0 = \frac{w_1 l^3}{360EI}\frac{l}{l_1}\left\{8 - 21\frac{l_1}{l} + 30\left(\frac{l_1}{l}\right)^2 - 10\left(\frac{l_1}{l}\right)^3\right\}.$$

The problem is essentially solved hereby. To obtain the desired formula for y_B, we need only to substitute the above expression for θ_0 into the equation for y_x and put $x = l_1$, getting thus

$$y_B = \frac{w_0 l_1 l^3}{360EI}\left\{8 - 13\frac{l_1}{l} + 7\left(\frac{l_1}{l}\right)^2 + 12\left(\frac{l_1}{l}\right)^3 - 6\left(\frac{l_1}{l}\right)^4\right\}.$$

5. BEAMS OF VARYING CROSS-SECTION

Here we must first recall that the fundamental equation for the bending of beams, $d^2y/dx^2 = -M/EI$, obtained in Eq. *c*, is valid at every point along the bent beam, irrespective of whether its flexural stiffness EI is constant or a function of x. In the latter case, assuming that I varies with x, the above relation may be written in the form

$$\frac{d^2y}{dx^2} = -\frac{1}{EI_0}\frac{M}{I/I_0} = -\frac{1}{EI_0}\bar{M}.$$

Here $\bar{M}$ stands for a moment diagram that is corrected by the variable I/I_0 ratios along the beam. From the above equality it is obvious that this $\bar{M}$ distribution of bending moment will produce exactly the same deflection line in a beam of constant EI_0 flexural rigidity, as the original M moment diagram would have in the original beam of varying moment of inertia.

A graphical representation of this concept is shown in Fig. 9*a*, where a simply supported beam is assumed to have along the *ab* portion a

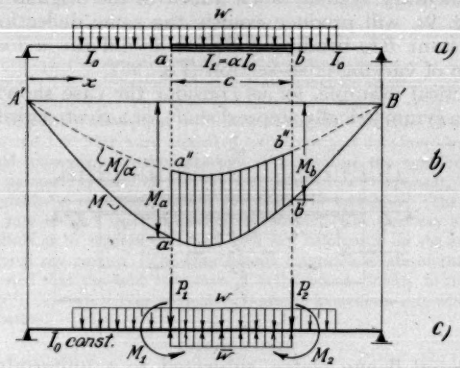

FIG. 9.

larger moment of inertia, $I_1 = \alpha I_0$, than the constant I_0 for the rest of the beam. The M diagram in this case is represented by the continuous curve $A'a'b'B'$ in Fig. 9*b*, while the $\bar{M} = M/(I/I_0)$ diagram differs from this by the amount of the shaded area $a'a''b''b'$, which corresponds to the reduction in the original M diagram due to dividing it by the factor $\alpha = I/I_0 > 1$.

The same reduction in the original M diagram can be obtained, however, in another manner. Namely, we can add to the original loading of the beam a complementary system of loading which will produce a negative moment area $a'a''b''b'$, thus correcting the original M diagram in the same way as the increase in I did along the *ab* portion. In every case, when the desired reduction in the M diagram is given, the complementary loading accomplishing this can be easily found from conditions of statics. In Fig. 9, for instance, we find that the complementary system will consist of a loading

$$\bar{w} = w\left(1 - \frac{1}{\alpha}\right)$$

uniformly distributed along the *ab* length of the beam, in addition to two concentrated moments (M_1, M_2) and two concentrated forces

(P_1, P_2) acting at the end points of this region, where

$$M_1 = M_a\left(1 - \frac{1}{\alpha}\right), \quad M_2 = M_b\left(1 - \frac{1}{\alpha}\right)$$

and

$$P_1 = \frac{M_1 - M_2}{c} - \frac{\overline{w}c}{2}; \quad P_2 = -\frac{M_1 - M_2}{c} - \frac{\overline{w}c}{2}.$$

This complementary system, when added to the original w loading, as shown in Fig. 9*c*, will produce exactly the same deflection curve in a beam of constant EI_0 bending stiffness, as did the w loading on the original beam of varying cross-section (Fig. 9*a*).

As a practical example, let us consider the case shown in Fig. 10, representing a symmetrically stepped shaft, or a riveted girder with sym-

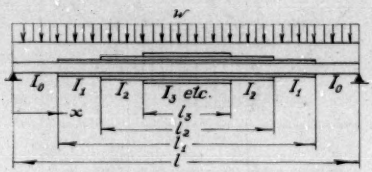

FIG. 10.

metrically placed flange plates, subjected to a uniformly distributed loading. It can be shown by the above described procedure, that the maximum deflection of a beam of this type, for any number of steps in its moment of inertia, can be calculated from the following formula

$$y_{x=\frac{l}{2}} = \frac{wl^4}{384EI_0}\left[5 - \sum_{n=0,1,2,3\ldots}\left(\frac{I_0}{I_n} - \frac{I_0}{I_{n+1}}\right)\frac{l_{n+1}}{l}\left\{3\left(\frac{l_{n+1}}{l}\right)^3 - 4\left(\frac{l_{n+1}}{l}\right)^2 - 6\frac{l_{n+1}}{l} + 12\right\}\right].$$

6. SUMMARY

It has been shown that the elastic lines of bent beams can be expressed by mean of Maclaurin series, each term of which is defined by one of the initial conditions of the beam. Though the method involves no more approximation than the ones ordinarily made in structural mechanics, and its results are therefore exact in this sense of the word, it is faster and more convenient than any of those in use at present. It can be employed equally well in the solution of statically determinate or indeterminate cases, including discontinuous loadings or variable cross-sections, and it has further possibilities in the analysis of elastic stability and vibration problems.

RADIATIONS FROM Zr^{97} AND Nb^{97} *

BY

C. E. MANDEVILLE,[1] E. SHAPIRO,[1] R. I. MENDENHALL,[1] E. R. ZUCKER[2] AND G. L. CONKLIN[1]

ABSTRACT

$Zr^{96}O_2$ (isotopic concentration 90 per cent) was irradiated by slow neutrons in the Oak Ridge pile. The half-period of the niobium daughter element, chemically separated from zirconium, was found to be 72.1 ± 0.7 min. and that of Zr^{97} to be 17.0 ± 0.2 hr. By aluminum absorption and Feather analysis, maximum beta ray enerieqs of 2.50 Mev and 1.40 Mev were measured for parent and daughter element, respectively. Lead absorption of the quantum radiations of the equilibrium mixture indicated a gamma ray at 0.74 Mev as well as a softer component. Coincidence absorption yielded a maximum gamma ray energy of 1.42 Mev. The beta-gamma coincidence rate of Nb^{97} was constant, independent of the beta ray energy and of such magnitude as to suggest that each beta ray is followed on the average by 0.7 Mev of gamma ray energy. The beta-gamma coincidence rate of the equilibrium mixture showed that the hard beta rays of Zr^{97} proceed directly to the metastable state of Nb^{97}. Very few beta rays of Zr^{97} are in immediate coincidence with any gamma radiation.

INTRODUCTION

The properties of the 17-hr. Zr^{97} and of its daughter element, the 70-min. Nb^{97}, have already been the subject of considerable investigation (1–5).[3] Absorption techniques have been employed to show that the beta rays of Zr^{97} have a maximum energy of 2.2 Mev and those of Nb^{97} a maximum energy of 1.4 Mev (3). The gamma rays of either isotope were measured by lead absorption to be approximately 0.8 Mev (3), and it was concluded that in the case of both actitivies, each beta ray is accompanied on the average by one gamma ray. Subsequent spectrometric measurements (5) have yielded beta ray energies of 1.91 ± 0.02 Mev and 1.267 ± 0.02 Mev and gamma ray energies of 0.747 ± 0.005 Mev for Zr^{97} and 0.665 ± 0.005 Mev for Nb^{97}. The gamma ray at 0.747 ± 0.005 Mev was shown to be emitted from an isomeric level in Nb^{97} of half period 60 sec. (5).

In the present investigation, $Zr^{96}O_2$ (isotopic concentration 90 per cent in Zr^{96}), obtained from the Y-12 plant, Carbide and Carbon Chemicals Division, Union Carbide and Carbon Corportion, Oak Ridge, Tennessee, was irradiated by slow neutrons in the Oak Ridge pile. The

* Assisted by the joint program of the ONR and the AEC.

[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] Frankford Arsenal, Philadelphia, Pa.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

radioactive materials were received within twenty-four hours after cessation of irradiation and chemical separations were immediately commenced.

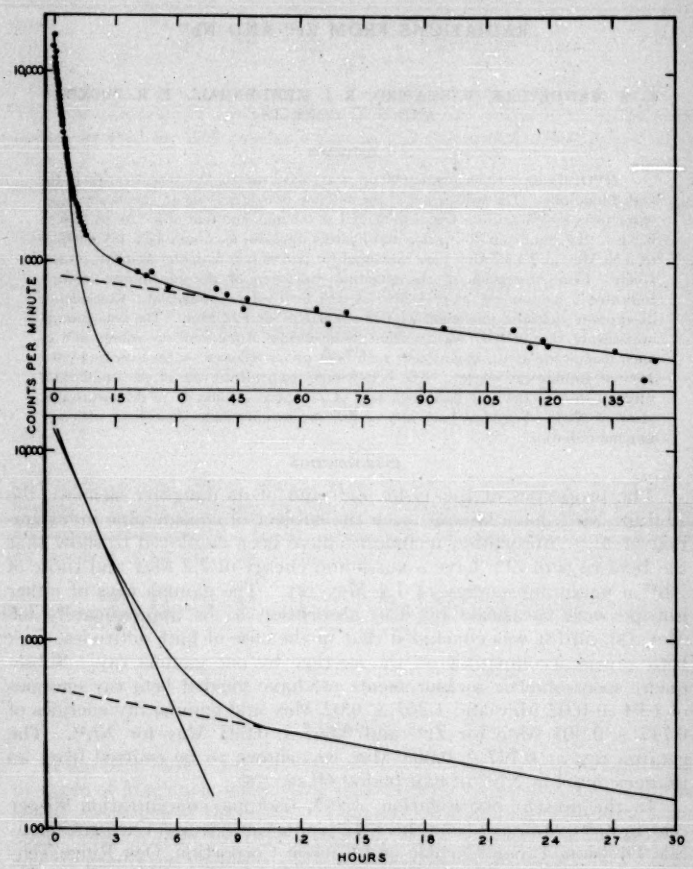

FIG. 1. Decay of the 72-min. Nb^{97}.

The slow neutron irradiated zirconium dioxide was dissolved by potassium pyrosulfate fusion. The separation of the niobium daughter activity from zirconium was effected by the use of Steinberg's "oxalate" procedure (6). A crystalline precipitate, presumably potassium acid

oxalate, formed on addition of excess oxalic acid to the aqueous solution of the fusion mixture. An assay, made after centrifugation, showed that most of the activity remained in solution. The centrifugate was poured off and retained. The activity carried with the precipitate was

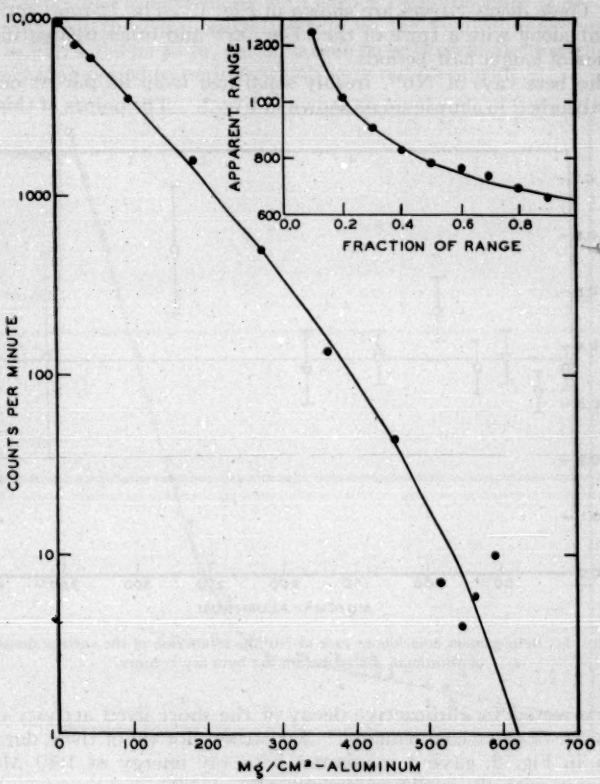

FIG. 2. Absorption in aluminum of the beta rays of Nb^{97}.

recovered by dissolving the precipitate in hot water, recrystallizing it from cold water, centrifuging, and combining the centrifugates. The separation outlined above was carried out on several successive occasions to supply fresh sources of Nb^{97}.

Nb^{97}

The decay of Nb^{97}, freshly separated from its parent element, was followed for ten half-periods, and the half-period, taken from the slope of the decay curve was found to be 72.1 ± 0.7 min. This value is to be compared with previously reported values of 68 min. (7) and 75 min. (2). These decay curves are shown in Fig. 1. The 72-min. activity is present along with a trace of the 17-hr. Zr^{97} and other unidentified impurities of longer half-periods.

The beta rays of Nb^{97}, freshly separated from its parent element, were absorbed in aluminum as shown in Fig. 2. The points of this curve

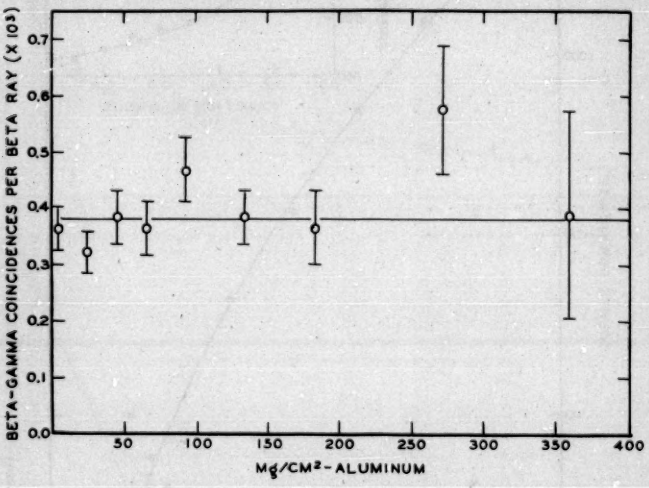

FIG. 3. Beta-gamma coincidence rate of Nb^{97} as a function of the surface density of aluminum placed before the beta ray counter.

were corrected for radioactive decay of the short lived activity during the course of the measurements. A Feather plot (8) of these data, also shown in Fig. 2, gave a maximum beta ray energy of 1.40 Mev, in agreement with the earlier measurements.

A third source of Nb^{97} was prepared to obtain the beta-gamma coincidence rate of Fig. 3. It is seen to be constant, independent of the beta ray energy, suggesting that the beta ray spectrum of Fig. 2 is a simple one. Calibration of the beta-gamma coincidence counting arrangement by the beta-gamma coincidence rate of Sc^{46} showed that each beta ray of Nb^{97} is followed, on the average, by 0.7 Mev of gamma

ray energy. Each point of Fig. 3 was, of course, properly corrected for decay of the source.

Zr^{97}

The radioactive decay of Zr^{97}–Nb^{97} was followed for ~200 hr. and the resulting half-period for the parent element, Zr^{97}, was calculated to be 17.0 ± 0.2 hr., in agreement with earlier measurements (2,3). The decay curve of the equilibrium mixture of the two radio-elements is shown in Fig. 4. The 17-hr. period is seen to be present along with some relatively long period impurities which were not identified.

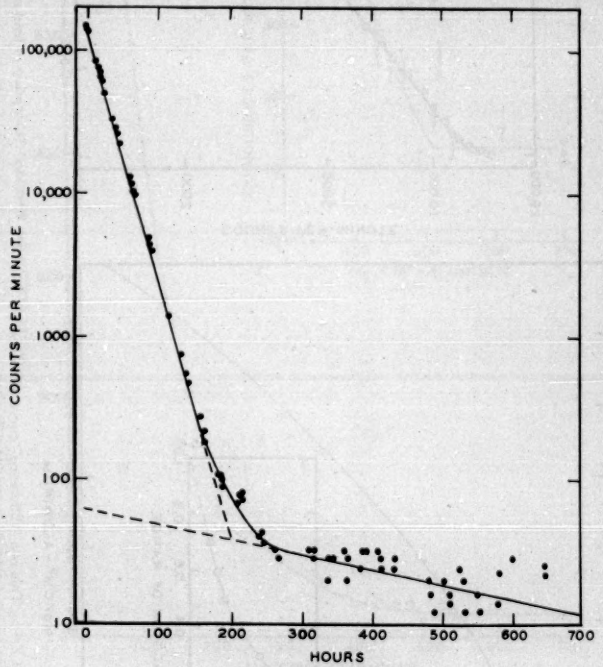

FIG. 4. Decay of the 17-hr. Zr^{97}.

The beta rays of the equilibrium mixture of the two activities was absorbed in aluminum. The assumption was made that at zero absorber thickness, half of the counting rate could be assigned to the beta rays of the 72-min. Nb^{97} and half to Zr^{97}. Assuming this equality of contribution at zero absorber thickness, the beta spectrum of Nb^{97} was

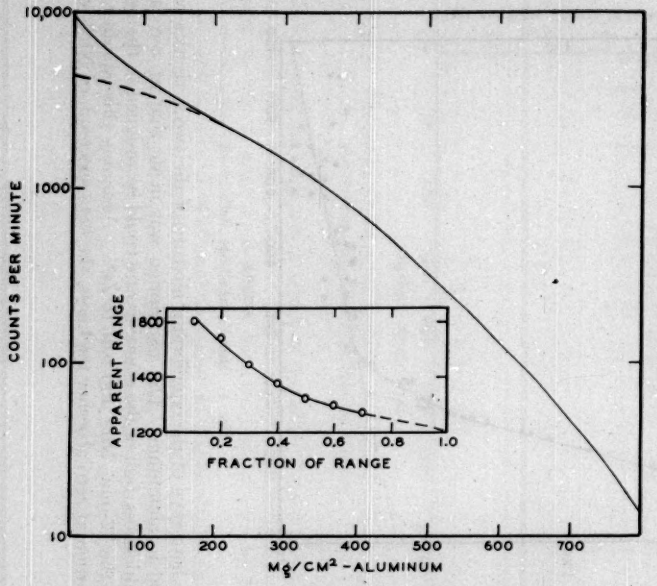

FIG. 5. Beta spectrum of Zr^{97}. This is a "difference" curve, obtained by subtracting the spectrum of Nb^{97} from that of Zr^{97}–Nb^{97}.

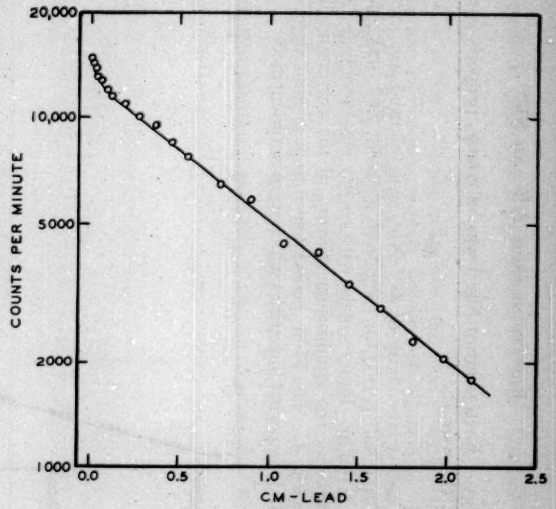

FIG. 6. Absorption in lead of the quantum radiations of Zr^{97}–Nb^{97}.

subtracted from the beta spectrum of the equilibrium mixture to give a difference spectrum of maximum energy 2.50 Mev. A semi-logarithmic plot of these beta rays also showed that a softer spectrum (end point 200 mg./cm.^{2}) is also present in the decay of Zr^{97}. These absorption data are plotted in Fig. 5.

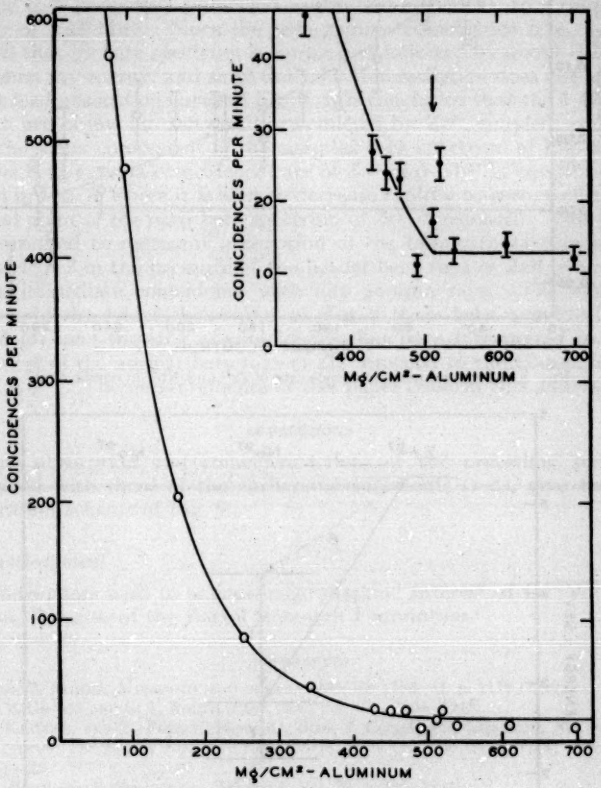

FIG. 7. Coincidence absorption of the secondary electrons of the gamma rays of Zr^{97}.

The gamma rays of the equilibrium mixture of the two activities was absorbed in lead as shown in Fig. 6, and the quantum energy taken from the slope of the curve was found to be 0.74 Mev. From the initial points of the curve, it is evident that a softer component is also present.

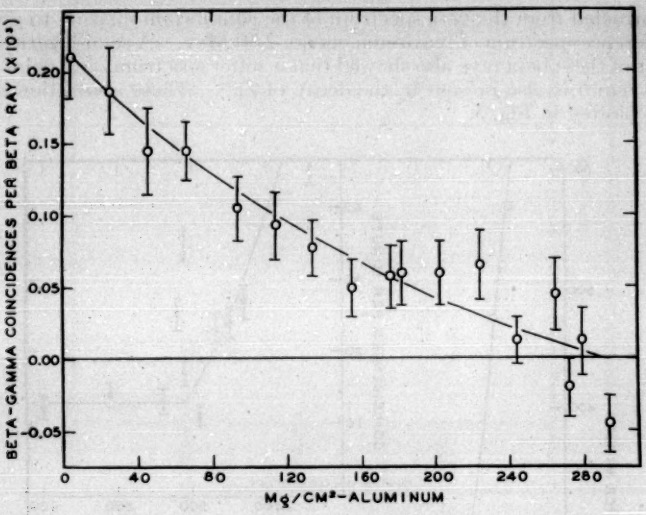

FIG. 8. Beta-gamma coincidence rate of Zr^{97} and Nb^{97} in equilibrium.

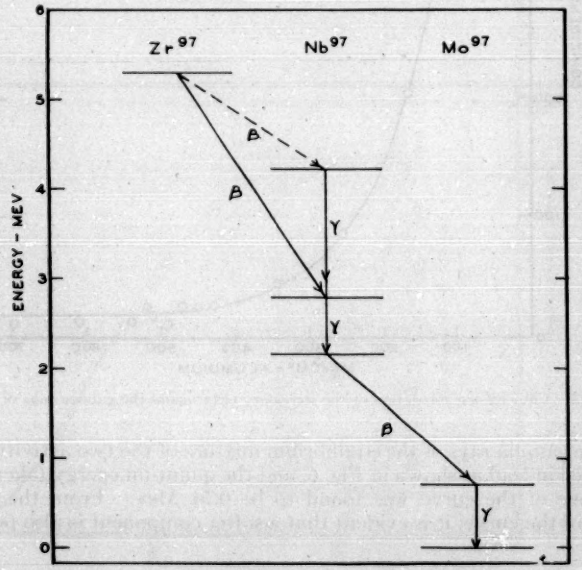

FIG. 9. Disintegration scheme for $Zr \xrightarrow{97\beta} Nb \xrightarrow{97\beta} Mo^{97}$

To measure the energy of the most energetic gamma-ray emitted in the decay of Zr^{97}-Nb^{97}, a source was placed on an aluminum block before two coincident thin-walled G-M tubes. A coincidence absorption curve of the secondary electrons ejected from the aluminum block is shown in Fig. 7. The end point of the distribution corresponds, according to a previously published calibration curve (9), to a quantum energy of 1.42 Mev. Since the beta-gamma coincidence rate of Nb^{97} showed that its beta spectrum is simple and followed by about 0.7 Mev of gamma ray energy, and since the 1.42 Mev radiation does not appear on the lead absorption curve of Fig. 6, it is concluded that the 1.42 Mev quanta are of low intensity and are emitted by Zr^{97}, coupled, perhaps, with the softer component of the complex beta spectrum of Fig. 5.

The beta-gamma coincidence rate of Zr^{97} and Nb^{97} in equilibrium is shown in Fig. 8, where it is seen to decrease rapidly to zero, well before the end point of the hard beta spectrum of Zr^{97} is reached. This curve is interpreted to represent absorption of the beta-gamma coincidence rate of Fig. 3 in the presence of the harder beta rays of Zr^{97}, which are not in immediate coincidence with any gamma rays. The essential features of the coincidence curve of Fig. 8 have been previously observed (5), and the lack of coincidences has been interpreted to show that most of the nuclear beta rays of Zr^{97} proceed to the 60-sec. isomer (5) of Nb^{97}. The measurements of this paper confirm this analysis.

CONCLUSIONS

The absorption and coincidence data of the preceding sections, combined with those of the earlier measurements (1–5), give the disintegration scheme of Fig. 9.

Acknowledgment

The writers wish to acknowledge the kind interest of Dr. W. F. G. Swann, Director of the Bartol Research Foundation.

REFERENCES

(1) SAGANE, KOJIMA, MIYAMOTO, AND IKAWA, *Phys. Rev.*, Vol. 57, p. 1179 (1940).
(2) A. V. GROSSE AND E. T. BOOTH, *Phys. Rev.*, Vol. 57, p. 664 (1940).
(3) S. KATCOFF, AND B. FINKLE, Paper 83, Book 2, Coryell and Sugarman, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Co., Inc., New York, New York), p. 705.
(4) O. HAHN AND F. STRASSMAN, *Naturwiss.*, Vol. 29, p. 285 (1941).
(5) BURGUS, KNIGHT, AND PRESTWOOD, *Phys. Rev.*, Vol. 79, p. 104 (1950).
(6) E. P. STEINBERG, Paper 243, Book 3, Coryell and Sugarman, *Radiochemical Studies: The Fission Products* (McGraw-Hill Book Co., Inc., New York, New York), p. 1495.
(7) G. E. BOYD, communication to Seaborg and Perlman, *Rev. Mod. Phys.*, Vol. 20, p. 585 (1948).
(8) N. FEATHER, *Proc. Camb. Phil. Soc.*, Vol. 30, p. 599 (1938).
(9) C. E. MANDEVILLE AND M. V. SCHERB, *Phys. Rev.*, Vol. 73, p. 1434 (1948).

Personal Lightning Rods?—Do you need a personal lightning rod to protect yourself from lightning?

A speaker at the Centennial of Engineering observance is regularly the target of such questions from the curious and concerned. He is Dr. Karl B. McEachron, sometimes known as "Mr. Lightning" and manager of the General Electric Company's Laboratory-Engineering Department at Pittsfield, Mass.

One person who feared that lightning might strike him while riding horseback asked if a chain dangled from his head to the ground would offer protection. He got the idea from watching trucks and their trailing chains, he said.

Dr. McEachron pointed out to this person that it was more probable he would get a "jolt" from the chain catching on some object than it was likely that he would be "jolted" by lightning.

Recently the scientist was asked about the soundness of a "lightning rod halo" a man had invented to protect himself against static electricity. As described by the inventor, the "personal lightning rod" consisted of a ring with a spike sticking up from it. This was to be worn on the head. The inventor claimed he had obtained very good results in reducing the static electricity with his device.

"Wouldn't it be a good idea to offer this device on the commercial market?" the inventor asked.

Dr. McEachron's reply to this person summarily was that the static electricity could better be eliminated by raising the moisture content of the room rather than by awkward bothersome gadgets for the person.

While the lightning authority acknowledges that lightning can be dangerous to persons, he tells those seeking his advice that in most cases "the cure is worse than the disease."

"The two safest places in the home during an electrical storm are the refrigerator and the furnace," he declares, adding hastily, "But who wants to burn up or freeze to death?"

To have protection in a backless and bootless bed, one only needs to attach rods at each corner of the bed, Mr. McEachron explains.

"But who wants to have anything like this in his home?" he asks.

The G-E scientist and his associates for many years have studied lightning, both natural and manmade, to provide more protection for electric systems and to improve electric power equipment.

He states in a paper written for a centennial symposium, "The source of serious interruption to the operation of power systems has been substantially brought under control. Enough knowledge is now available so that a high degree of immunity from lighting can be obtained."

The first testing of high voltage power transformers with laboratory lightning, the development of shielded transformers and lightning arresters that perform dependably, are all results of General Electric's high voltage research that contribute to today's continuity of electric service, he says, reviewing progress in the electrical field.

For the future Dr. McEachron says, "More work is yet to be done in the area of reducing costs and improving performance of apparatus involved, and a more detailed knowledge of lightning itself will be helpful."

His paper, entitled "Lightning—A Hazard To Electric Systems," was given at the Power Division Symposium on Sept. 11.

COMPENSATION OF FEEDBACK-CONTROL SYSTEMS SUBJECT TO SATURATION *

BY

GEORGE C. NEWTON, JR.[1]

Part II [2]

V. EXAMPLES

A. Background for First Two Examples

The major objective of this research is to develop a design procedure for compensating networks which takes cognizance of the possibility that certain components of a physical network may impose limitations because of their finite ranges of linearity. Up to the present time network synthesis theory usually has presupposed components which are linear over infinite ranges. It was left to the application engineer to select physical parts with sufficiently wide ranges of linearity so that the designed system would not operate with any part in the saturation region. For much engineering work this is a satisfactory procedure. However, in certain applications, particularly in automatic-control applications, this procedure could result in requiring excessively large components, since component operating range is allowed to play little part in establishing the design. In certain situations it would seem possible to reduce linear range requirements by permitting the system error to increase slightly from the minimum value possible with linear range considerations excluded from the system determination. One can conceive of a "trading curve" for system design which governs the exchange of reduction of linear range requirements for increase in system error. If specifications are such that the "trading curve" permits a large reduction of linear range for a small increase in error, one is inclined to make use of this fact in making his design. This is particularly

* This paper is abstracted from the theoretical portion of a thesis presented to the Masssachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Science in Electrical Engineering. The basic problem of dealing with saturation in feedback-control systems has arisen repeatedly in the author's experience in the automatic control field. A partial solution was obtained during work for the Fairchild Guided Missile Division, Fairchild Engine and Airplane Corp., under Navy Contract NOa(s) 9020. The more general solution contained in this paper was obtained in the course of research conducted in the Servomechanisms Laboratory at M.I.T. under Air Force Contract W33-038ac-13969.

The author is indebted to Drs. G. S. Brown, A. C. Hall and Y. W. Lee of the Massachusetts Institute of Technology for their sponsorship of this research. He wishes to thank his colleagues at the Massachusetts Institute of Technology for their constructive criticism of the research and its reportings.

[1] Assistant Professor of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] Part I appeared in this JOURNAL for October, 1952.

true in servomechanisms where the size of the servomotor is a measure of system size and cost in weight and money.

The first two examples to follow seek to establish the "trading curves" relating linear range requirement to error for two sets of specifications differing only in the variable considered to be critical as to linear range. The specifications for these examples have been purposely selected to make the analytical work as simple as possible and still illustrate the fact that "trading-curve" shape favorable to reduction of linear range may or may not exist, depending upon the problem specifications. In all examples, the rms. values of the saturation signals are considered to be a measure of the linear range requirement.

B. Example 1

Given a servomechanism problem in which the input is pure message and the desired output is just the input message. Given further that the fixed elements of the problem impose no limitations other than the possibility of output *position* saturation. Consider the input to be a stationary, stochastic function characterized by the autocorrelation function:

$$\phi_{ii}(\tau) = \sigma_{ip}^2 e^{-|f\tau|}. \tag{38}$$[3]

Find the optimum system function as a function of the limit on the rms. value of the saturation signal. Determine the function relating the limit on the rms. saturation signal to the rms. error for the optimum system.

For this problem, since the desired output is the same as the input,

$$\Phi_{ii}(\omega) = \Phi_{id}(\omega). \tag{39}$$

Now from the definition of the input power-density spectrum

$$\Phi_{ii}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{-j\omega\tau} \sigma_{ip}^2 e^{-|f\tau|}, \tag{40}$$

or

$$\Phi_{ii}(\omega) = \frac{\sigma_{ip}^2}{\pi} \frac{f}{\omega^2 + f^2}. \tag{41}$$

Also for this problem $H_f(\omega)$ and $H_{s1}(\omega)$ can be chosen to be

$$H_f(\omega) = 1 \tag{42}$$

$$H_{s1}(\omega) = 1 \tag{43}$$

since the fixed elements are premised to impose no limitations other than possible output-position saturation.

[3] Such an autocorrelation function could result from an input function which is constant except at change points having a Poisson distribution in time. The constant amplitudes between changes could be governed by a normal distribution of standard deviation σ_{ip}.

In the general solution formula (Eq. 23), the values of $A(\omega)$ and $B(\omega)$ from Eqs. 26 and 27, respectively, are

$$A(\omega) = (1 + \rho_{s1}) \frac{\sigma_{ip}^2 f}{\pi} \frac{1}{\omega^2 + f^2} \tag{44}$$

$$B(\omega) = \frac{\sigma_{ip}^2 f}{\pi} \frac{1}{\omega^2 + f^2} \tag{45}$$

since only one point of saturation is involved and no transient specification is imposed.

We may take

$$A^-(\omega) = \frac{1}{-j\omega + f} \tag{46}$$

and

$$A^+(\omega) = \frac{(1 + \rho_{s1})\sigma_{ip}^2 f}{\pi} \frac{1}{j\omega + f}. \tag{47}$$

Accordingly,

$$\frac{B(\omega)}{A^-(\omega)} = \frac{\sigma_{ip}^2 f}{\pi} \frac{1}{j\omega + f} \tag{48}$$

and

$$\left[\frac{B(\omega)}{A^-(\omega)}\right]_+ = \frac{\sigma_{ip}^2 f}{\pi} \frac{1}{j\omega + f}. \tag{49}$$

Thus

$$H_{c0}(\omega) = \frac{\pi L(j\omega + f)}{(1 + \rho_{s1})\sigma_{ip}^2 f} + \frac{1}{1 + \rho_{s1}}. \tag{50}$$

Because the rms. level of the output (and also of the error) would approach infinity if L were different from zero, it is evident that L must be zero. Since $H_f(\omega)$ is unity, the over-all optimum system function $H_0(\omega)$ is equal to $H_{c0}(\omega)$. Thus

$$H_0(\omega) = \frac{1}{1 + \rho_{s1}}, \tag{51}$$

which is a simple attenuator.

Having found the proper form of $H_0(\omega)$, it is now necessary to eliminate ρ_{s1} and find $H_0(\omega)$ in terms of the rms. saturation signal σ_{s1}. In this case, since the output is the saturation signal,

$$\sigma_{s1}^2 = \int_{-\infty}^{\infty} d\omega \bar{H}_0(\omega) H_0(\omega) \Phi_{ii}(\omega). \tag{52}$$

Integration yields

$$\sigma_{s1}^2 = \frac{\sigma_{ip}^2}{(1 + \rho_{s1})^2}. \tag{53}$$

$H_0(\omega)$ in terms of σ_{s1} is then simply

$$H_0(\omega) = \frac{\sigma_{s1}}{\sigma_{ip}} \quad \text{for} \quad 0 \leqslant \sigma_{s1} \leqslant \sigma_{ip}. \tag{54}$$

Denoting the limit upon the rms. saturation signal as σ_{sm1}, it is evident that $H_0(\omega)$ should be

$$\begin{aligned} H_0(\omega) &= \frac{\sigma_{sm1}}{\sigma_{ip}} \quad \text{for} \quad \sigma_{sm1} \leqslant \sigma_{ip} \\ &= 1 \quad \text{for} \quad \sigma_{sm1} \geqslant \sigma_{ip} \end{aligned} \tag{55}$$

for minimum rms. error. This is the first result asked for.

In order to obtain the second required result, the rms. error σ_e must be found in terms of σ_{sm1}, using the system function (Eq. 55). Because the input and desired output are identical, the minimum mean-square error is, in this case,

$$\sigma_e^2 = \int_{-\infty}^{\infty} d\omega[\overline{1 - H_0(\omega)}][1 - H_0(\omega)]\Phi_{ii}(\omega) \tag{56}$$

or

$$\begin{aligned} \sigma_e^2 &= \left(1 - \frac{\sigma_{sm1}}{\sigma_{ip}}\right)^2 \sigma_{ip}^2 \quad \text{for} \quad \sigma_{sm1} \leqslant \sigma_{ip} \\ &= 0 \quad \text{for} \quad \sigma_{sm1} \geqslant \sigma_{ip}. \end{aligned} \tag{57}$$

Hence

$$\begin{aligned} \sigma_e &= \left(1 - \frac{\sigma_{sm1}}{\sigma_{ip}}\right) \sigma_{ip} \quad \text{for} \quad \sigma_{sm1} \leqslant \sigma_{ip} \\ &= 0 \quad \text{for} \quad \sigma_{sm1} \geqslant \sigma_{ip} \end{aligned} \tag{58}$$

is the second required result.

The foregoing results may be expressed in another form as

$$\begin{aligned} H_0(\omega) &= S_{m1} \quad \text{for} \quad S_{m1} \leqslant 1 \\ &= 1 \quad \text{for} \quad S_{m1} \geqslant 1 \end{aligned} \tag{59}$$

$$\begin{aligned} E &= 1 - S_{m1} \quad \text{for} \quad S_{m1} \leqslant 1 \\ &= 0 \quad \text{for} \quad S_{m1} \geqslant 1, \end{aligned} \tag{60}$$

where S_{m1} is the limit on the rms. saturation signal expressed in normalized form as

$$S_{m1} = \frac{\sigma_{sm1}}{\sigma_{ip}}. \tag{61}$$

E is the rms. error expressed in normalized form as

$$E = \frac{\sigma_e}{\sigma_{ip}}. \tag{62}$$

Figure 6 shows how the limit on the saturation signal level varies as a function of the allowable error for this simple example. It is evident

from this "trading curve" that the cost of reducing the saturation signal level, as measured in terms of error, is so great that once the "natural" saturation limit of unity is passed, it is not feasible to trade error for reduction of linear range. This is what one would intuitively surmise in this example. All that has been done is to quantitatively confirm intuition.

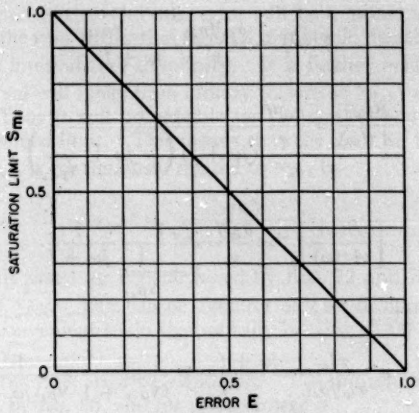

FIG. 6. Saturation limit *versus* error for example 1.

C. Example 2

This example is identical to the first example, except that the fixed elements impose no limitations other than the possibility of output *velocity* saturation (as distinguished from output *position* saturation in Example 1.)

Here again $\Phi_{ii}(\omega) = \Phi_{id}(\omega)$, $\Phi_{ii}(\omega) = [\sigma^2_{ip}/\pi][f/(\omega^2 + f^2)]$ and $H_f(\omega) = 1$. To account for output velocity saturation, take

$$H_{s1}(\omega) = j\omega. \tag{63}$$

Then by Eqs. 26 and 27

$$A(\omega) = \frac{\sigma_{ip}^2 f}{\pi}\left[\frac{1}{\omega^2 + f^2} + \frac{\rho_{s1}\omega^2}{\omega^2 + f^2}\right] \tag{64}$$

$$B(\omega) = \frac{\sigma_{ip}^2 f}{\pi}\frac{1}{\omega^2 + f^2}. \tag{65}$$

Write

$$A(\omega) = \frac{\sigma_{ip}^2 f \rho_{s1}}{\pi}\frac{\omega^2 + \dfrac{1}{\rho_{s1}}}{\omega^2 + f^2} \tag{66}$$

and factor into

$$A^-(\omega) = \frac{-j\omega + \frac{1}{\sqrt{\rho_{s1}}}}{-j\omega + f} \tag{67}$$

$$A^+(\omega) = \frac{\sigma_{ip}^2 f \rho_{s1}}{\pi} \frac{j\omega + \frac{1}{\sqrt{\rho_{s1}}}}{j\omega + f}. \tag{68}$$

Then

$$\frac{B(\omega)}{A^-(\omega)} = \frac{\sigma_{ip}^2 f}{\pi} \frac{1}{(j\omega + f)\left(-j\omega + \frac{1}{\sqrt{\rho_{s1}}}\right)} \tag{69}$$

and

$$\left[\frac{B(\omega)}{A^-(\omega)}\right]_+ = \frac{\sigma_{ip}^2 f}{\pi} \cdot \frac{1}{f + \frac{1}{\sqrt{\rho_{s1}}}} \cdot \frac{1}{j\omega + f}. \tag{70}$$

Thus, by Eq. 23,

$$H_{c0}(\omega) = \frac{\pi L}{\sigma_{ip}^2 f \rho_{s1}} \cdot \frac{j\omega + f}{j\omega + \frac{1}{\sqrt{\rho_{s1}}}} + \frac{1}{f\sqrt{\rho_{s1}} + 1} \cdot \frac{1}{\sqrt{\rho_{s1}} j\omega + 1}. \tag{71}$$

L must be zero for finite rms. output velocity. Also H_f is unity. Therefore, the optimum over-all system function is

$$H_0(\omega) = \frac{1}{f\sqrt{\rho_{s1}} + 1} \cdot \frac{1}{\sqrt{\rho_{s1}} j\omega + 1}. \tag{72}$$

In this example, the saturation signal is the output velocity, and its mean square value is given by

$$\sigma_{s1}^2 = \int_{-\infty}^{\infty} d\omega (-j\omega) \bar{H}_0(\omega) (j\omega) H_0(\omega) \Phi_{ii}(\omega) \tag{73}$$

or

$$\sigma_{s1}^2 = \frac{\sigma_{ip}^2 f}{\pi} \frac{1}{(f\sqrt{\rho_{s1}} + 1)^2} \int_{-\infty}^{\infty} d\omega \frac{\omega^2}{\bar{D}(\omega) D(\omega)}, \tag{74}$$

where $D(\omega) = \sqrt{\rho_{s1}}(j\omega)^2 + (f\sqrt{\rho_{s1}} + 1)j\omega + f$. Evaluation of the integral (James, Nichols, and Phillips, (1),[4] has an appendix which is useful

[4] The boldface numbers in parentheses refer to the references appended to Part I of this paper, see this JOURNAL, October, 1952, p. 296.

in this regard) gives

$$\sigma_{s1}^2 = \frac{\sigma_{ip}^2 f}{\sqrt{\rho_{s1}}(f\sqrt{\rho_{s1}} + 1)^2}. \tag{75}$$

Algebraic complication prevents solving this last equation for $\sqrt{\rho_{s1}}$ and thereby obtain an explicit expression for $H_0(\omega)$ in terms of ρ_{s1}. However, it is apparent that the rms. error will be a monotonic increasing function and the rms. saturation signal a monotonic decreasing function of $\sqrt{\rho_{s1}}$ in the interval zero to infinity. It is further evident that rms. saturation signal will range from infinity to zero as $\sqrt{\rho_{s1}}$ varies from zero to infinity. Thus it will be possible for the rms. saturation signal to reach any specified limit. This means that the limit on the saturation signal level σ_{sm1} is continuously related to $\sqrt{\rho_{s1}}$ by

$$\sigma_{sm1}^2 = \frac{\sigma_{ip}^2 f}{\sqrt{\rho_{s1}}(f\sqrt{\rho_{s1}} + 1)^2}. \tag{76}$$

The first result asked for is determined by Eqs. 72 and 76 through the elimination of $\sqrt{\rho_{s1}}$. This can be done for any set of numerical values, but it is not convenient to do algebraically because of the fourth degree equation for $\sqrt{\rho_{s1}}$.

Next, let the rms. error be evaluated. The minimum mean-square error is given by Eq. 56, and for this example is

$$\sigma_e^2 = \frac{\sigma_{ip}^2 f}{\pi}\int_{-\infty}^{\infty} d\omega \left[1 - \frac{1}{f\sqrt{\rho_{s1}} + 1}\right] \left[1 - \frac{1}{\sqrt{\rho_{s1}}(-j\omega) + 1}\right] \times \left[1 - \frac{1}{f\sqrt{\rho_{s1}} + 1}\right]\left[\frac{1}{\sqrt{\rho_{s1}}j\omega + 1}\right]\left[\frac{1}{\omega^2 + f^2}\right] \tag{77}$$

or

$$\sigma_e^2 = \frac{\sigma_{ip}^2 f \rho_{s1}}{\pi}\int_{-\infty}^{\infty} d\omega \frac{\omega^2 + \dfrac{f^2}{(f\sqrt{\rho_{s1}} + 1)^2}}{D(\omega)D(\omega)}, \tag{78}$$

where $D(\omega) = \sqrt{\rho_{s1}}(j\omega)^2 + (f\sqrt{\rho_{s1}} + 1)j\omega + f$. Integration yields

$$\sigma_e^2 = \sigma_{ip}^2 \frac{f\sqrt{\rho_{s1}}(f\sqrt{\rho_{s1}} + 1)^2 + (f\sqrt{\rho_{s1}})^2}{(f\sqrt{\rho_{s1}} + 1)^3}. \tag{79}$$

As there is no direct way of eliminating $\sqrt{\rho_{s1}}$ between Eqs. 79 and 76, these two equations together constitute the second required result.

The results of Example 2 may be summarized in slightly different

form as

$$H_0(\omega) = \frac{1}{\alpha + 1} \frac{1}{\frac{\alpha}{f} j\omega + 1} \tag{80}$$

$$S_{m1} = \frac{1}{\sqrt{\alpha}(\alpha + 1)^{\frac{1}{2}}} \tag{81}$$

$$E = \frac{\sqrt{\alpha(\alpha + 1)^2 + \alpha^2}}{(\alpha + 1)^{\frac{1}{2}}} \tag{82}$$

where

$$\alpha = f\sqrt{\rho_{s1}} \tag{83}$$

$$S_{m1} = \frac{\sigma_{sm1}}{\sigma_{ip}f} \tag{84}$$

$$E = \frac{\sigma_e}{\sigma_{ip}}. \tag{85}$$

Curve *A* in Fig. 7 is for this example what Fig. 6 is for the first example. Note that here the "trading curve" is so shaped as to favor reduction of the limit on the saturation signal level through increasing the error, since the cost in error is not so great as in Example 1. In-

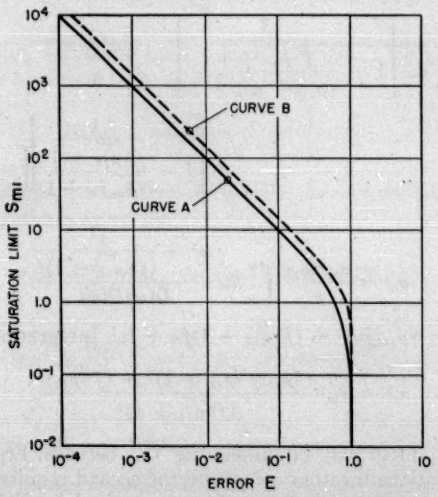

FIG. 7. Saturation limit *versus* error for example 2.

deed, in view of these results, a designer facing the specifications of this problem would be foolish not to reduce the burden on his servomotor by allowing a little error to occur. Although it is very simple and the results may seem extreme, this example appears to justify the "hunch" that advantages may be gained by injecting saturation considerations into a system design problem at the outset rather than at the end of its solution.

At this point the reader may ask, "How sensitive are these results to departures of the system function from its optimum form?" Some insight into this question may be gained by comparing results for a particular nonoptimum system with those above for the optimum system. For example, let the nonoptimum over-all system function be

$$H(\omega) = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\xi\frac{j\omega}{\omega_n} + 1} \tag{86}$$

instead of the optimum function given by Eq. 72.

By expressing the limit on the saturation signal level σ_{sm1} and the rms. error σ_e in terms of the parameters of $H(\omega)$, and by expressing the results nondimensionally, one obtains

$$S_{m1} = \sqrt{\frac{1}{2\xi\alpha(1 + 2\xi\alpha + \alpha^2)}} \tag{87}$$

$$E = \sqrt{\frac{\alpha\left[\left(2\xi + \frac{1}{2\xi}\right) + \alpha\right]}{1 + 2\xi\alpha + \alpha^2}} \tag{88}$$

(both equations for α such that $E \leqslant 1.0$), where

$$\alpha = \frac{f}{\omega_n} \tag{89}$$

$$S_{m1} = \frac{\sigma_{sm1}}{\sigma_{ip}f} \tag{84}$$

$$E = \frac{\sigma_e}{\sigma_{ip}}. \tag{85}$$

Curve B in Fig. 7 is a plot S_{m1} *versus* E for ξ equal to one-half. Notice that the same general relation is established between saturation limit and error for the nonoptimum system as for the optimum system; the only practical difference is that in the low error range about 40 per cent more error is necessary to obtain the same saturation limit.

D. Background for Third Example

Sometimes the designer is given rather meager information about the message component of the input to a system. This may be for lack of knowledge of the statistical properties of this component, or it may be because the actual message is not describable in terms which fit into the known schemes of analysis. Under such conditions it is not uncommon to specify the transient response of the system to some degree as a substitute for input message specification. In addition, certain of the system error coefficients (defined in reference (1)) may be specified.

With respect to the noise component of the input to a system, frequently it is found that noise predominates over message in producing saturation effects. For example, in a radar tracking device, noise can become the predominant cause of saturation. Thus, even though the message component of the input need not be directly accounted for, the noise may very well have to be considered in a system design.

The following example considers a problem where the input noise is assumed to be the major cause of saturation, and where the input message is not specifiable and is therefore assumed to be zero. In lieu of the input message specification, requirements on the position-error coefficient and the transient response are laid down.

E. Example 3

Given a servomechanism problem in which the normal input is pure noise (and the desired output is therefore zero). Given that the transient response of the servomechanism must be such that the integral-square error I_{e1} to a step-function input is equal to or less than a finite value I_{em1} to be determined on the basis that the input step is the desired output. (This automatically makes the position-error coefficient zero.) Consider the input noise to be "white noise" characterized by the autocorrelation function

$$\phi_{nn}(\tau) = \frac{2\eta\delta(\tau)}{(1 \text{ unit of time})}, \tag{90}$$

where $\delta(\tau)$ is the unit impulse function. Given further that the fixed elements of the problem impose no limitations other than the possibility of output *velocity* saturation.

First, find the optimum system function as a function of the limit on the rms. saturation signal and the limit on the transient error. Second, find the relationship among rms. error, saturation limit, and transient-error limit when the optimum system function is used.

For this problem, $\Phi_{ii}(\omega) = \Phi_{nn}(\omega)$, and therefore

$$\Phi_{ii}(\omega) = \frac{\eta}{\pi}. \tag{91}$$

Because the desired output is zero,

$$\Phi_{id}(\omega) = 0. \tag{92}$$

In order to handle the transient constraint, the translation functions, $I_{iik}(\tau)$ and $I_{idk}(\tau)$, have to be transformable (see Eqs. 15 and 17 for definitions), and this fact will prevent direct use of step functions. However, let the input transient be

$$C_{i1}(t) = Vu(t)e^{-at}, \tag{93}$$

where $u(t)$ is the unit step function, and a is a positive number which can approach zero in the limit (e^{-at} may be thought of as a convergence factor). Then from the definition (Eq. 15)

$$I_{ii1}(\tau) = V^2 \int_{-\infty}^{\infty} dt u(t)e^{-at}u(t+\tau)e^{-a(t+\tau)} \tag{94}$$

or

$$I_{ii1}(\tau) = \frac{V^2 e^{-a|\tau|}}{2a} \tag{95}$$

and

$$I_{ii1}(\omega) = \frac{V^2}{2\pi}\frac{1}{\omega^2 + a^2}. \tag{96}$$

Because the input and desired output are the same for the transient-response test

$$I_{id1}(\omega) = I_{ii1}(\omega). \tag{97}$$

$H_f(\omega)$ and $H_{s1}(\omega)$ for this problem may be taken as

$$H_f(\omega) = 1 \tag{98}$$

$$H_{s1}(\omega) = j\omega. \tag{99}$$

Referring to the solution form (Eq. 23), the values of $A(\omega)$ and $B(\omega)$ from Eqs. 26 and 27, respectively, are

$$A(\omega) = \frac{\eta}{\pi} + \frac{\rho_{c1}V^2}{2\pi}\cdot\frac{1}{\omega^2 + a^2} + \frac{\rho_{s1}\eta}{\pi}\omega^2 \tag{100}$$

$$B(\omega) = \frac{\rho_{c1}V^2}{2\pi}\cdot\frac{1}{\omega^2 + a^2}. \tag{101}$$

Rewrite $A(\omega)$ as

$$A(\omega) = \frac{\rho_{s1}\eta}{\pi}\frac{\omega^4 + \left(a^2 + \frac{1}{\rho_{s1}}\right)\omega^2 + \frac{a^2}{\rho_{s1}} + \frac{\rho_{c1}V^2}{2\rho_{s1}\eta}}{\omega^2 + a^2}. \tag{102}$$

Factor $A(\omega)$ into

$$A^-(\omega) = \frac{(-j\omega + c + jd)(-j\omega + c - jd)}{-j\omega + a} \tag{103}$$

$$A^+(\omega) = \frac{\rho_{s1}\eta}{\pi} \frac{(j\omega + c + jd)(j\omega + c - jd)}{j\omega + a}, \tag{104}$$

where

$$c = \sqrt{\frac{1}{4}\left(a^2 + \frac{1}{\rho_{s1}}\right) + \frac{1}{2}\sqrt{\frac{a^2}{\rho_{s1}} + \frac{\rho_{c1}V^2}{2\rho_{s1}\eta}}} \tag{105}$$

$$d = \sqrt{\frac{1}{2}\sqrt{\frac{a^2}{\rho_{s1}} + \frac{\rho_{c1}V^2}{2\rho_{s1}\eta}} - \frac{1}{4}\left(a^2 + \frac{1}{\rho_{s1}}\right)}. \tag{106}$$

Hence

$$\frac{B(\omega)}{A^-(\omega)} = \frac{\rho_{c1}V^2}{2\pi} \frac{1}{(j\omega + a)(-j\omega + c + jd)(-j\omega + c - jd)} \tag{107}$$

and

$$\left[\frac{B(\omega)}{A^-(\omega)}\right]_+ = \frac{\rho_{c1}V^2}{2\pi(a^2 + 2ca + c^2 + d^2)} \cdot \frac{1}{j\omega + a}. \tag{108}$$

Thus

$$H_{c0}(\omega) = \frac{\pi L}{\rho_{s1}\eta} \frac{j\omega + a}{(j\omega + c + jd)(j\omega + c - jd)} + \frac{\rho_{c1}V^2}{2\rho_{s1}\eta(a^2 + 2ca + c^2 + d^2)} \cdot \frac{1}{(j\omega + c + jd)(j\omega + c - jd)}. \tag{109}$$

L must be zero since, if it were not, the saturation signal level would be infinite. Therefore, $H_0(\omega)$, which is the same as $H_{c0}(\omega)$ in this problem where $H_f(\omega) = 1$, is

$$H_0(\omega) = \frac{\rho_{c1}V^2}{2\rho_{s1}\eta(a^2 + 2ca + c^2 + d^2)(c^2 + d^2)} \cdot \frac{1}{\frac{(j\omega)^2}{c^2 + d^2} + \frac{2c}{c^2 + d^2}j\omega + 1}. \tag{110}$$

Consider now the limit of $H_{c0}(\omega)$ as a approaches zero.

$$\lim_{a \to 0} (c^2 + d^2) = \sqrt{\frac{\rho_{c1}V^2}{2\rho_{s1}\eta}} \tag{111}$$

$$\lim_{a \to 0} \frac{2c}{c^2 + d^2} = \sqrt{\frac{2\eta}{\rho_{c1}V^2}} + 2\sqrt{\frac{2\rho_{s1}\eta}{\rho_{c1}V^2}}. \tag{112}$$

Let the following abbreviations be used

$$b_2 = \sqrt{\frac{\rho_{c1} V^2}{2\rho_{s1}\eta}} \tag{113}$$

$$b_1 = \sqrt{\frac{2\eta}{\rho_{c1} V^2}} + 2\sqrt{\frac{2\rho_{s1}\eta}{\rho_{c1} V^2}}. \tag{114}$$

Then the limit of $H_0(\omega)$ becomes

$$\frac{1}{b_2(j\omega)^2 + b_1 j\omega + 1}. \tag{115}$$

This is the optimum form of the over-all system function for limited integral-square error to step-function disturbances and limited velocity saturation signal when the normal input is white noise only. This form will be used from here on to the end of this problem.

The integral-square error I_{s1} for the step input $Vu(t)$ is now evaluated in terms of the parameters of $H_0(\omega)$. A corollary of the real convolution theorem states that

$$\int_{-\infty}^{\infty} dt f^2(t) = 2\pi \int_{-\infty}^{\infty} d\omega \bar{F}(\omega) F(\omega). \tag{116}$$

Letting $[1 - H_0(\omega)]V/2\pi j\omega = F(\omega)$ in Eq. 116 gives

$$I_{s1} = \frac{V^2}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{b_2^2\omega^2 + b_1^2}{\bar{D}(\omega) D(\omega)}, \tag{117}$$

where $D(\omega) = b_2(j\omega)^2 + b_1 j\omega + 1$. Integrating yields

$$I_{s1} = \frac{V^2}{2}\left(\frac{b_2}{b_1} + b_1\right). \tag{118}$$

The rms. saturation-signal level σ_{s1} is next evaluated.

$$\sigma_{s1}^2 = \frac{\eta}{\pi} \int_{-\infty}^{\infty} d\omega \frac{\omega^2}{\bar{D}(\omega) D(\omega)}. \tag{119}$$

Hence

$$\sigma_{s1} = \sqrt{\frac{\eta}{b_1 b_2}}. \tag{120}$$

Finally, the rms. error σ_e is determined.

$$\sigma_e^2 = \frac{\eta}{\pi} \int_{-\infty}^{\infty} d\omega \frac{1}{\bar{D}(\omega) D(\omega)}, \tag{121}$$

where $D(\omega) = b_2(j\omega)^2 + b_1(j\omega) + 1$. Therefore

$$\sigma_e = \sqrt{\frac{\eta}{b_1}}. \tag{122}$$

Inspection of Eqs. 118, 120, and 122 indicates that the error can always be minimized by pushing the saturation level σ_{s1} up to its prescribed limit σ_{sm1} and by pushing the integral-square error I_{e1} for a step input up to its prescribed limit I_{em1}. Therefore, the first result asked for may be summarized as

$$H_0(\omega) = \frac{1}{b_2(j\omega)^2 + b_1 j\omega + 1} \tag{115}$$

$$I_{em1} = \frac{V^2}{2}\left(\frac{b_2}{b_1} + b_1\right) \tag{123}$$

$$\sigma_{sm1} = \sqrt{\frac{\eta}{b_1 b_2}}. \tag{124}$$

These equations relate the optimum system function to the limits set upon the integral-square transient error and the rms. saturation signal. It is not convenient to solve algebraically for the b's because of the cubic equation involved; numerical solutions for particular cases can always be handled, however.

The second required result, the relationship among error, saturation limit, and transient error limit is contained in Eqs. 122, 123, and 124. Eliminating b_2 and b_1 yields

$$\left(\frac{\sigma_e^2}{\eta}\right)^3 + \frac{\sigma_{sm1}^2}{\eta}\left(1 - \frac{2I_{em1}}{V^2}\frac{\sigma_e^2}{\eta}\right) = 0 \tag{125}$$

or

$$\frac{\sigma_{sm1}^2}{\eta} = \frac{\left(\frac{\sigma_e^2}{\eta}\right)^3}{\frac{2I_{em1}}{V^2}\frac{\sigma_e^2}{\eta} - 1}. \tag{126}$$

This result may be expressed in another form as

$$S_{m1} = \frac{E^3}{\sqrt{2Q_{m1}E^2 - 1}}, \tag{127}$$

where

$$S_{m1} \triangleq \frac{\sigma_{sm1}}{\sqrt{\eta}}. \tag{128}$$

$$E \triangleq \frac{\sigma_e}{\sqrt{\eta}} \tag{129}$$

$$Q_{m1} \triangleq \frac{I_{em1}}{V^2}. \tag{130}$$

Since Q_{m1} is normally the quantity which is most likely to be specified in advance and held constant throughout a problem, it is convenient for

plotting purposes to write Eq. 127 as

$$Q_{m1}^{\frac{1}{2}}S_{m1} = \frac{(Q_{m1}^{\frac{1}{2}}E)^3}{\sqrt{2(Q_{m1}^{\frac{1}{2}}E)^2 - 1}}. \tag{131}$$

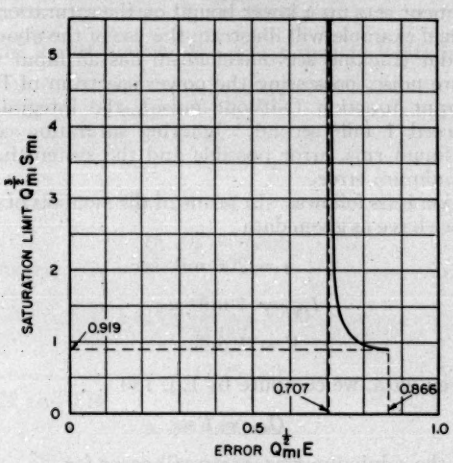

FIG. 8. Saturation limit *versus* error for example 3.

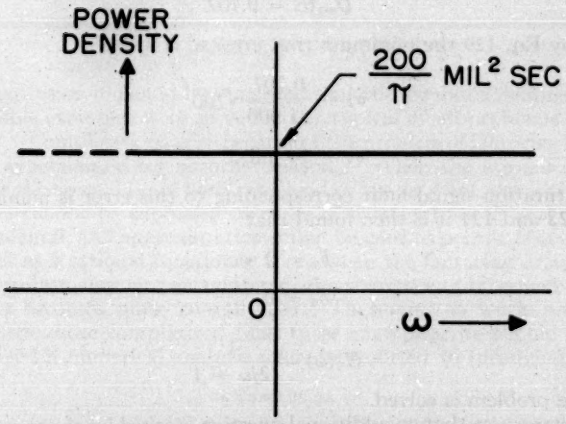

FIG. 9. Input power density spectrum for example 3.

Figure 8 is a plot showing how the saturation-signal limit varies with error for this problem. Note that only a small increase in the rms. error

of about 23 per cent will permit a reduction of the saturation-signal limit from infinity to less than one of the non-dimensional units. Note further that it is impossible to reduce the saturation-signal limit much below unity on the scale used. This is because the integral-square error requirement sets up a lower bound on the saturation limit.

A numerical example will illustrate the use of the above results. A simplified radar tracking servomechanism has an input which is substantially pure noise, possessing the power spectrum of Fig. 9. For a 1-mil step-input function (without noise), the integral-square error shall not exceed 1 mil² second. Ignoring saturation considerations, find the minimum rms. error possible and the system function which yields this minimum error.

The solution is as follows. In terms of the symbols of the preceding paragraphs, we have as given data

$$\eta = 200 \text{ mil}^2 \text{ sec.} \tag{132}$$

$$I_{em1} = 1 \text{ mil}^2 \text{ sec.} \tag{133}$$

$$V = 1 \text{ mil.} \tag{134}$$

From the given data, we compute by Eq. 130

$$Q_{m1} = 1 \text{ sec.}$$

From Fig. 8, the minimum rms. error will occur for

$$Q_{m1}^{\frac{1}{2}} E = 0.707. \tag{135}$$

Thus by Eq. 129 the minimum rms. error σ_e is

$$\sigma_e = \frac{0.707}{\sqrt{1.000}} \sqrt{200}$$

$$\sigma_e = 10 \text{ mils.} \tag{136}$$

The saturation-signal limit corresponding to this error is infinite. By Eqs. 122 and 124 it is then found that

$$b_1 = 2 \text{ sec.} \tag{137}$$

$$b_2 = 0. \tag{138}$$

Thus

$$H_0(\omega) = \frac{1}{2j\omega + 1} \tag{139}$$

and the problem is solved.

Suppose now that an additional question is asked for the above servomechanism. What is the lowest limit that can be imposed upon the rms. output velocity; what is the minimum rms. error when this limit

is imposed; and what is the optimum system function under these conditions?

Solving, we have by Fig. 8 that the lowest saturation-signal limit occurs for

$$Q_{m1}^{\frac{1}{2}}S_{m1} = 0.918. \tag{140}$$

By Eq. 128, the lowest limit on the rms. output velocity is

$$\sigma_{\dot{s}m1} = \frac{0.918\sqrt{200}}{1^{\frac{1}{2}}} \tag{141}$$

$$\sigma_{\dot{s}m1} = 13 \text{ mil sec.}^{-1}.$$

Also from Fig. 8 we note that for the lowest saturation-signal limit

$$Q_{m1}^{\frac{1}{2}}E = 0.866 \tag{142}$$

so that the rms. error is

$$\sigma_e = \frac{0.866}{\sqrt{1.000}}\sqrt{200}$$

$$\sigma_e = 12.25 \text{ mil.} \tag{143}$$

Using Eqs. 122 and 124

$$b_1 = 1.333 \text{ sec.} \tag{144}$$

$$b^2 + 0.888 \text{ sec.}^2 \tag{145}$$

Hence

$$H_0(\omega) = \frac{1}{0.888(j\omega)^2 + 1.333j\omega + 1}. \tag{146}$$

In conclusion it should be remarked that the algebraic neatness with which these examples were solved is not typical of all problems met in practice. Complications arise because of the problem of factoring higher-order polynomials, if for no other reason. When the normal desired output is the normal input with pure lead or lag, or when the correlation functions cannot be expressed as sums of exponentials, $H_{c0}(\omega)$ will be transcendental, and approximations must be used to permit $H_{c0}(\omega)$ to be expressed as a rational function. Even when the factoring or approximation problems are not encountered, the structure of intermediate expressions becomes quite complicated. Therefore, to work problems only a little more complicated than these examples, we should not be discouraged if numerical analysis must be resorted to throughout.

VI. CONCLUSION

This paper has presented a theory for the design of compensating networks in filters and feedback-control systems. In formulating this theory, which is based upon the work of Wiener and Lee, an attempt is

made to recognize the possibility of saturation in the fixed components of the system and, in addition, to consider the transient performance of the over-all system. Both the saturation and the transient performance considerations have been omitted in the previous generalized theories of filter and feedback-control system design in spite of the fact that these considerations are of great practical import.

The treatment for compensating-network design used in this paper developed from two fundamental ideas. The first is that, pending the development of a generalized approach to nonlinear system design, the practical designer is interested in building systems in which the saturable components are operated substantially in their linear ranges so that the over-all performance can be predicted on the basis of linear theory. This leads to the principle of designing for a limited probability of saturation. If it is assumed that the probability of saturation of a physical system can be controlled by controlling the rms. values of the saturation signals, that would exist in a linear model of such a system, then it is shown that a fairly simple mathematical formulation of the problem is possible.

The second fundamental idea is that possibilities for the control of transient performance exist if the integral-square errors to a set of transient test signals can be controlled. It is shown to be entirely feasible to incorporate such control into the mathematical formulation of the compensating-network design problem.

The assumptions made on the basis of the two fundamental ideas concerning saturation and transient performance, together with those used by Wiener and Lee, have been shown to lead to a rather general solution of the compensating-network design problem. This solution has been demonstrated for several specific examples showing that concrete results can be obtained by means of it.

The results of this paper should be helpful to servomechanism designers, especially in evaluating the ultimate limitations placed upon a system's performance by its fixed components such as its servomotor. Although it has been long recognized that limits exist upon what can be accomplished by means of compensation devices for systems such as servomechanisms, little quantitatively exact knowledge of these limits has been available. It is hoped that the methods of this paper will help to improve this situation.

Designers of active networks for filters and other applications may find the methods outlined above useful in their problems when large latitude with respect to saturation or transient behavior is not allowed by the problem specifications.

It is regretted that all the results of this paper have to be premised upon the rms. error as a criterion of system goodness. Certain extensions to other criteria may be possible, but these all would appear to have at least as limited usefulness as the rms. criterion.

APPENDIX

VII. DERIVATION OF RESULTS

A. Expression for Rms. Error σ_e

This section gives the detailed steps by which the solution for the compensating-network transfer relationships are derived. In the broad outline of the procedure discussed in Subsection IV *B*, the first step was to obtain an expression for the rms. error σ_e in terms of the normal-input and desired-output correlation functions and the compensating-and fixed-network impulse responses. This is explained in detail in the following paragraphs.

First we need an expression for $\theta_0(t)$.

Referring to Fig. 5, we see that

$$\theta_f(t-y) = \int_{-\infty}^{\infty} dx h_c(x)\theta_i(t-x-y) \tag{147}$$

$$\theta_0(t) = \int_{-\infty}^{\infty} dy h_f(y)\theta_f(t-y). \tag{148}$$

Substituting the value of $\theta_f(t-y)$ of Eq. 147 into Eq. 148 yields

$$\theta_0(t) = \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_c(x)\theta_i(t-x-y). \tag{149}$$

From the definition of the error $\theta_e(t)$ given by Fig. 5 (*viz.*: $\theta_e(t) = \theta_d(t) - \theta_0(t)$), we write

$$\theta_e^2(t) = \theta_d^2(t) - 2\theta_d(t)\theta_0(t) + \theta_0^2(t). \tag{150}$$

From Eq. 149 for $\theta_0(t)$, we write for $\theta_0^2(t)$

$$\theta_0^2(t) = \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_c(x)\theta_i(t-x-y) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_c(u)\theta_i(t-u-v). \tag{151}$$

Equation 150 we now write as

$$\theta_e^2(t) = \theta_d^2(t) - 2\theta_d(t) \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_c(x)\theta_i(t-x-y) + \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_c(x)\theta_i(t-x-y) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_c(u)\theta_i(t-u-v). \tag{152}$$

The square of the rms. value σ_e of the error $\theta_e(t)$ is by definition

$$\sigma_e^2 = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \theta_e^2(t). \tag{153}$$

Substituting the value $\theta_e^2(t)$ of Eq. 152 into Eq. 153 yields upon interchanging orders of integration and limit processes

$$\sigma_e^2 = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \theta_d^2(t) - 2 \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_c(x) \lim_{T\to\infty} \int_{-T}^{T} dt \theta_i(t-x-y)\theta_d(t) + \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_c(x) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_c(u) \times \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \theta_i(t-x-y)\theta_i(t-u-v). \tag{154}$$

Recall that the definition of the autocorrelation function of any function $f_1(t)$ is

$$\phi_{f_1 f_1}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt f_1(t) f_1(t+\tau) \tag{155}$$

and that of the cross-correlation function of any pair of functions $f_1(t)$ and $f_2(t)$ is

$$\phi_{f_1 f_2}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt f_1(t) f_2(t+\tau). \tag{156}$$

Then Eq. 154 can be written in terms of auto- and cross-correlation functions of $\theta_i(t)$ and $\theta_d(t)$ as

$$\sigma_e^2 = \phi_{dd}(0) - 2\int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_e(x) \theta_{id}(x+y) + \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_e(x) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_e(u) \theta_{ii}(x+y-u-v). \quad (157)$$

(The "θ's" in the subscripts for the correlation functions have been omitted for simplicity, since no confusion is caused thereby.) Equation 157 is the desired expression for the rms. error σ_e in terms of the normal-input and desired-output correlation functions and the compensating- and fixed-network impulse responses.

B. Expression for Rms. Saturation Signal σ_{sn}

In order to obtain the necessary expression for the nth rms. saturation signal σ_{sn} in terms of the input and desired-output correlation functions and the compensating-network and saturation-signal computer impulse responses, we first write an expression for the nth saturation signal $\theta_{sn}(t)$. Figure 5 shows that this will be the same as the expression for $\theta_0(t)$ (Eq. 149) with $h_{sn}(t)$ substituted for $h_f(t)$. Thus

$$\theta_{sn}(t) = \int_{-\infty}^{\infty} dy h_{sn}(y) \int_{-\infty}^{\infty} dx h_e(x) \theta_i(t-x-y). \quad (158)$$

Hence

$$\theta_{sn}^2(t) = \int_{-\infty}^{\infty} dy h_{sn}(y) \int_{-\infty}^{\infty} dx h_e(x) \theta_i(t-x-y) \int_{-\infty}^{\infty} dv h_{sn}(v) \int_{-\infty}^{\infty} du h_e(u) \theta_i(t-u-v). \quad (159)$$

Now

$$\sigma_{sn}^2 \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \theta_{sn}^2(t). \quad (160)$$

Substituting the value of $\theta_{sn}^2(t)$ from Eq. 159 into Eq. 160, interchanging orders of integration and limit processes, and using the definition of the autocorrelation function yields

$$\sigma_{sn}^2 = \int_{-\infty}^{\infty} dy h_{sn}(y) \int_{-\infty}^{\infty} dx h_e(x) \int_{-\infty}^{\infty} dv h_{sn}(v) \int_{-\infty}^{\infty} du h_e(u) \phi_{ii}(x+y-u-v). \quad (161)$$

This is the desired expression for σ_{sn}.

C. Expression for Integral Square I_{ek} of Transient Error $C_{ek}(t)$

Next we find the expression for the kth integral-square error. The transient error $C_{ek}(t)$ is given by Eq. 152 with $C_{ek}^2(t)$ replacing $\theta_e^2(t)$, $C_{dk}(t)$ replacing $\theta_d(t)$, and $C_{ik}(t)$ replacing $\theta_i(t)$.

$$C_{ek}^2(t) = C_{dk}^2(t) - 2C_{dk}(t) \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_e(x) C_{ik}(t-x-y) + \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_e(x) C_{ik}(t-x-y) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_e(u) C_{ik}(t-u-v). \quad (162)$$

Use of the definition of the integral-square transient error I_{ek} (Eq. 20) and of the definitions of the translation functions I_{iik}, I_{idk} (Eqs. 15, 17, respectively) yields

$$I_{ek} = I_{ddk}(0) - 2\int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_e(x) I_{idk}(x+y) + \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_e(x) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_e(u) I_{iik}(x+y-u-v) \quad (163)$$

when integration orders are interchanged. $I_{ddk}(\tau)$ is defined as

$$I_{ddk}(\tau) \triangleq \int_{-\infty}^{\infty} dt C_{dk}(t) C_{dk}(t+\tau). \quad (164)$$

Equation 163 gives the desired expression for the integral-square error I_{ek} in terms of the transient-input and desired-output translation functions and the impulse responses of the compensating and fixed networks.

D. Derivation of the Integral Equation 22

Equations 157, 161, and 163 give the rms. error σ_e, the N rms. values σ_{sn} of the saturation signals, and the K integral-square values I_{ek} of the transient errors as functions of $h_c(t)$. The mathematical problem is to find the curve $h_c(t)$ which minimizes σ_e and which at the same time satisfies the $N + K$ conditions $\sigma_{sn} \leqslant \sigma_{smn}$ and $I_{ek} \leqslant I_{emk}$. Also $h_c(t)$ must satisfy, of course, the condition that $h_c(t) = 0$ for $t < 0$ in order to ensure a realizable compensating network. Beyond these conditions $h_c(t)$ is unbounded.

The finding of $h_c(t)$ is effectively the solving of an isoperimetric problem in the calculus of variations. It can be shown that the $h_c(t)$ which minimizes the function

$$F[h_c(t)] \triangleq \sigma_e^2 + \sum_{n=1}^{N} \rho_{sn}\sigma_{sn}^2 + \sum_{k=1}^{K} \rho_{ek} I_{ek}, \tag{165}$$

where ρ_{sn} and ρ_{ek} are $N + K$ Lagrangian multipliers, is the required one (11, 12). (To be precise, the $h_c(t)$ which minimizes the function F of Eq. 165 is that which makes σ_e^2 a minimum within the $N + K$ constraints $\sigma_{sn} = \sigma_{spn}$ and $I_{ek} = I_{epk}$, where σ_{spn} and I_{epk} are numbers which are functions of the ρ_{sn} and ρ_{ek} multipliers. It is evident that this is the $h_c(t)$ which we desire, since suitable adjustment of the multipliers will ensure that the limit conditions on the rms. saturation signals and integral-square transient errors are met.)

The procedure for finding the $h_c(t)$ which minimizes the F of Eq. 165 is to assume that it exists. Denote this optimum value of $h_c(t)$ as $h_{c0}(t)$. Then consider an $h_c(t)$ different from this optimum formed by

$$h_c(t) = h_{c0}(t) + \epsilon h_{c1}(t), \tag{166}$$

where $h_{c1}(t)$ is any fixed, arbitrary, realizable impulse response and ϵ is a variable introduced for convenience. Use of this $h_c(t)$ in Eq. 165 will yield an F which is a function of ϵ. Since $h_{c0}(t)$ is the optimum, we know that $F(\epsilon)$ should have a minimum for $\epsilon = 0$. Thus a necessary condition on $h_{c0}(t)$ is obtained by setting

$$\frac{dF(\epsilon)}{d\epsilon} = 0 \quad \text{at} \quad \epsilon = 0. \tag{167}$$

This condition in conjunction with the limit conditions proves to be sufficient to determine $h_{c0}(t)$.

Consider now the components of F. The derivative of each one with respect to ϵ is to be determined and the results added in order to form Eq. 167. Differentiating Eq. 157 and setting ϵ equal to zero yields:

$$\begin{aligned}\left.\frac{d(\sigma_e^2)}{d\epsilon}\right|_{\epsilon=0} = &-2\int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_{c1}(x)\phi_{id}(x+y) \\ &+ \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_{c1}(x) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_{c0}(u)\phi_{ii}(x+y-u-v) \\ &+ \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dx h_{c0}(x) \int_{-\infty}^{\infty} dv h_f(v) \int_{-\infty}^{\infty} du h_{c1}(u)\phi_{ii}(x+y-u-v)\end{aligned} \tag{168}$$

because $\phi_{ii}(\tau)$ is an even function, the last two terms are equal. Making use of this fact and interchanging orders of integration gives

$$\begin{aligned}\left.\frac{d\sigma_e^2}{d\epsilon}\right|_{\epsilon=0} = &\ 2\int_{-\infty}^{\infty} dx h_{c1}(x) \int_{-\infty}^{\infty} du h_{c0}(u) \int_{-\infty}^{\infty} dy h_f(y) \int_{-\infty}^{\infty} dv h_f(v)\phi_{ii}(x+y-u-v) \\ &- 2\int_{-\infty}^{\infty} dx h_{c1}(x) \int_{-\infty}^{\infty} dy h_f(y)\phi_{id}(x+y).\end{aligned} \tag{169}$$

Equation 161 when differentiated and when ϵ is set equal to zero, results in a sum of two terms similar to the last two terms of Eq. 168. Making use of the fact that these terms are equal and interchanging order of integration yields

$$\left.\frac{d\sigma_{sn}^2}{d\epsilon}\right|_{\epsilon=0} = 2\int_{-\infty}^{\infty} dx h_{c1}(x) \int_{-\infty}^{\infty} du h_{c0}(u) \int_{-\infty}^{\infty} dy h_{sn}(y) \int_{-\infty}^{\infty} dv h_{sn}(v)\phi_{ii}(x+y-u-v). \tag{170}$$

Treating Eq. 163 in an identical fashion to Eq. 157 will give by analogy to Eq. 169

$$\frac{dI_{ak}}{d\epsilon}\bigg|_{\epsilon=0} = 2\int_{-\infty}^{\infty} dx h_{\epsilon 1}(x)\int_{-\infty}^{\infty} du h_{\epsilon 0}(u)\int_{-\infty}^{\infty} dy h_f(y)\int_{-\infty}^{\infty} dv h_f(v) I_{iik}(x+y-u-v) - 2\int_{-\infty}^{\infty} dx h_{\epsilon 1}(x)\int_{-\infty}^{\infty} dy h_f(y) I_{iik}(x+y). \quad (171)$$

Adding together the components of $\frac{dF(\epsilon)}{d\epsilon}\Big|_{\epsilon=0}$ (in accordance with Eq. 165) produces the following necessary condition:

$$2\int_{-\infty}^{\infty} dx h_{\epsilon 1}(x)[\text{Left side of Eq. 22}]. \quad (172)$$

But $h_{\epsilon 1}(x)$ is arbitrary except that it must be zero for $x < 0$. This means that the brackets must be zero for $x \geq 0$, since if they were not an $h_{\epsilon 1}(x)$ could be found which would invalidate Eq. 172. Thus Eq. 22 is shown to be a necessary condition on $h_{\epsilon 0}(t)$. (From what has been said so far, this could be a condition for either a maximum or minimum value of σ_ϵ^2. Consideration of $d^2F(\epsilon)/d\epsilon^2$ will show that we have a condition for a minimum since this quantity is guaranteed to be positive.) That Eq. 22 is a sufficient condition (in conjunction with the rms. saturation signal and integral-square-error limit conditions) to determine $h_{\epsilon 0}(t)$ is evident from the fact that unique solutions are obtained.

E. Derivation of the Solution Formula 23

The solution formula represented by Eq. 23 is obtained from the integral equation 22 by considering the Fourier transform of the left side of the latter. It is evident that the left side of Eq. 22 is a function of time which can only be non-zero for negative time; let this function of time be $l(x)$. Thus

$$l(x) = \int_{-\infty}^{\infty} du h_{\epsilon 0}(u)\left\{\begin{array}{l}\displaystyle\int_{-\infty}^{\infty} dy h_f(y)\int_{-\infty}^{\infty} dv h_f(v)[\phi_{ii}(x+y-u-v) \\ \displaystyle\quad + \sum_{k=1}^{K}\rho_{ak} I_{iik}(x+y-u-v)] + \sum_{n=1}^{N}\rho_{sn}\int_{-\infty}^{\infty} dy h_{sn}(y) \\ \displaystyle\qquad \times \int_{-\infty}^{\infty} dv h_{sn}(v)\phi_{ii}(x+y-u-v)\end{array}\right\}$$
$$- \int_{-\infty}^{\infty} dy h_f(y)[\phi_{id}(x+y) + \sum_{k=1}^{K}\rho_{ak} I_{iik}(x+y)]. \quad (173)$$

On the assumption that transforms exist for the correlation and translation functions, we can transform both sides of Eq. 173. This gives, upon interchanging orders of integration and rearranging,

$$l(\omega) = \int_{-\infty}^{\infty} du e^{-j\omega u} h_{\epsilon 0}(u)$$
$$\times\left\{\begin{array}{l}\displaystyle\int_{-\infty}^{\infty} dy e^{+j\omega y} h_f(y)\int_{-\infty}^{\infty} dv e^{-j\omega v} h_f(v)\,\frac{1}{2\pi}\left[\int_{-\infty}^{\infty} dx e^{-j\omega(x+y-u-v)}\phi_{ii}(x+y-u-v)\right. \\ \displaystyle\left.\quad + \sum_{k=1}^{K}\rho_{ak}\int_{-\infty}^{\infty} dx e^{-j\omega(x+y-u-v)} I_{iik}(x+y-u-v)\right] \\ \displaystyle\sum_{n=1}^{N}\rho_{sn}\int_{-\infty}^{\infty} dy e^{+j\omega y} h_{sn}(y)\int_{-\infty}^{\infty} dv e^{-j\omega v} h_{sn}(v)\,\frac{1}{2\pi}\int_{-\infty}^{\infty} dx e^{-j\omega(x+y-u-v)}\phi_{ii}(x+y-u-v)\end{array}\right\}$$
$$- \int_{-\infty}^{\infty} dy e^{+j\omega y}\,\frac{1}{2\pi}\left[\int_{-\infty}^{\infty} dx e^{-j\omega(x+y)}\phi_{id}(x+y) + \sum_{k=1}^{K}\rho_{ak}\int_{-\infty}^{\infty} dx e^{-j\omega(x+y)} I_{iid}(x+y)\right]. \quad (174)$$

Identifying the integrals in Eq. 174 as system functions, transforms of translation functions, and power-density spectra permits writing

$$I(\omega) = H_{e0}(\omega)\left\{\begin{array}{l}\bar{H}_f(\omega)H_f(\omega)[\Phi_{ii}(\omega) + \sum_{k=1}^{K} \rho_{ik}I_{iik}(\omega)] \\ \\ + \sum_{n=1}^{N} \rho_{in}\bar{H}_{in}(\omega)H_{in}(\omega)\Phi_{ii}(\omega)\end{array}\right\} - \bar{H}_f(\omega)[\Phi_{id}(\omega) + \sum_{k=1}^{K} \rho_{ik}I_{idk}(\omega)]. \quad (175)$$

(See Eqs. 33, 34, 35, and 36 for definitions of $\Phi_{ii}(\omega)$, $I_{iik}(\omega)$, $\Phi_{id}(\omega)$, and $I_{idk}(\omega)$.) Using the definitions of $A(\omega)$ and $B(\omega)$ given by Eqs. 26 and 27, respectively, Eq. 175 can be written as

$$I(\omega) = H_{e0}(\omega)A(\omega) - B(\omega). \quad (176)$$

We recognize that, since $i(t)$ is zero for positive time, $I(\omega)$ has no poles in the UHP. (UHP is an abbreviation for upper half plane; LHP, for lower half plane.) Let $A(\omega)$ be split into two factors: $A^+(\omega)$ with poles and zeros in the UHP only and $A^-(\omega)$ with poles and zeros in the LHP only, such that $A^+(\omega) \times A^-(\omega)$ is equal to $A(\omega)$. Divide both sides of Eq. 176 by $A^-(\omega)$ to obtain

$$\frac{I(\omega)}{A^-(\omega)} = H_{e0}(\omega)A^+(\omega) - \frac{B(\omega)}{A^-(\omega)}. \quad (177)$$

$\frac{I(\omega)}{A^-(\omega)}$ still has no poles in the UHP. Next let $B(\omega)/A^-(\omega)$ be separated into two parts; $\left[\frac{B(\omega)}{A^-(\omega)}\right]_+$ with poles in the UHP and $\left[\frac{B(\omega)}{A^-(\omega)}\right]_-$ with poles in the LHP, such that $\left[\frac{B(\omega)}{A^-(\omega)}\right]_+ + \left[\frac{B(\omega)}{A^-(\omega)}\right]_-$ is equal to $\frac{B(\omega)}{A^-(\omega)}$. Add to both sides of Eq. 177 the quantity $\left[\frac{B(\omega)}{A^-(\omega)}\right]_-$ to get

$$\frac{I(\omega)}{A^-(\omega)} + \left[\frac{B(\omega)}{A^-(\omega)}\right]_- = H_{e0}(\omega)A^+(\omega) - \left[\frac{B(\omega)}{A^-(\omega)}\right]_+. \quad (178)$$

The left side has no poles in the UHP. The right side as no poles in the LHP, since $H_{e0}(\omega)$ is a realizable system function. Thus each side of the equation is a function with no poles and is therefore constant. Let this constant be L. Then

$$L = H_{e0}(\omega)A^+(\omega) - \left[\frac{B(\omega)}{A^-(\omega)}\right]_+ \quad (179)$$

and rearranging gives the solution formula for $H_{e0}(\omega)$, Eq. 23.

It should be noted that the derivations in this section are purely formal and are not proof in the strict mathematical sense. Undoubtedly the above results could be proved rigorously by following the paths of Kolmogoroff (13) and Wiener (4).

Automatic Pouring of Molten Metals.—Ajax Engineering Corporation of Trenton, N. J., recently demonstrated the "Ajaxomatic," a new automatic unit for pouring accurate amounts of molten metal into molds.

The "Ajaxomatic" is designed to discharge molten metal, in particular aluminum, directly into the cold chamber of most available die casting machines. Molten metal is taken from a point considerably below the surface of the melt, eliminating any chance for pouring surface oxides or scum into the casting.

The "Ajaxomatic" acts as a combined holding furnace and automatic pouring unit. Temperature and alloy composition of the melt are maintained accurately by the well-known Ajax-Tama-Wyatt inductor in the lower part of the unit, based on the principle of gentle electromagnetic stirring. This unit is not a melting furnace. The "Ajaxomatic" is supplied with batches of molten aluminum from separate melting furnaces at regular intervals, thus fitting into the practice of most die casting plants where conventional hand ladling furnaces are in use.

The "Ajaxomatic" can also be used for zinc alloys, pointing the way to the economic use of cold chamber machines for that purpose, if desired.

Practically inert refractory materials and electric heating are used throughout. The molten aluminum is nowhere in contact with metals or moving parts.

The "Ajaxomatic" was designed especially to fit the requirements of aluminum die casting plants, based on years of experience in this industry with the Ajax Electromagnetic Pump.

Two entirely self-contained electric control unit sections form an important part of the complete "Ajaxomatic" installation. One section contains the conventional controls for the Ajax-Tama-Wyatt inductor and is used for accurately maintaining the metal temperature. The other section contains precision electronic timers and heating controls pertaining to the automatic pouring. The weight of the casting is selected on a dial in this section, and repeated automatically from then on. The normal range is for castings from ½ to 5 lb. Castings up to 20 lb. can be produced with slight modifications.

The "Ajaxomatic" controls can be linked to the automatic controls used on today's die casting machines. Thus, fully automatic operation can be obtained during each complete cycle of the die casting machine. With the time delay of ladling eliminated, production of aluminum die castings can be increased significantly, the operation being essentially as automatic as zinc die casting is today.

Furthermore, this linking of controls assures a constant time interval between pouring and injection, an important element in maintaining uniform casting quality.

The "Ajaxomatic" unit requires comparatively little space. The operator's side is left entirely clear.

The "Ajaxomatic" is even cooler than an induction holding furnace for hand ladling because it is completely enclosed. With the described arrangement, the machine operator will feel no heat at all.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

THERMOELECTRIC GENERATORS

It has been known for many decades that when two different metals are joined together, and the joint is heated, an electromotive force is developed. If the metals form a closed circuit, an electric current will flow.

Thermocouples and thermopiles have been widely used to measure temperature and to detect and measure radiant energy. Their utility depends in fact on their simplicity—no more convenient method for generating electromotive force or electric current by the absorption of thermal energy has ever been discovered. This very fact has for many years intrigued the minds of scientists and engineers intent on the development of new, improved or more convenient generators of electrical power.

A commercial thermoelectric generator would have many advantages. It would be quiet. It would have no moving parts and thus not require lubrication. Since the power conversion is so direct, the generator might be quite compact. It would be ideal for unattended operation in remote locations, and convenient for the transformation of atomic energy into electrical power.

Numerous patents have been granted in a number of countries for various thermoelectric generators. Such generators are not widely used because, up to now, their efficiency has been extremely low.

It has almost seemed that nature planned it that way, for whenever a material turns up with desirable properties it appears to have undesirable properties in about the same proportion. Desirable properties include high thermoelectric power, high electrical conductivity, and low thermal conductivity. In naturally occurring metals, high electrical conductivity is accompanied by high thermal conductivity with the result that excessive heat is lost by conduction. On the other hand, if the electrical conductivity is low, large voltage produces low electric current, with excessive heat losses in the leads.

During recent years, emphasis has been placed on research for new materials, or old materials in new forms. Semiconductors with trace impurities which radically change their properties, alloys, and intermetallic compounds give promise of sufficiently high efficiencies for certain practical uses. One such material is an intermetallic compound of zinc and antimony, which has been studied extensively by Dr. Maria

Telkes, of the Massachusetts Institute of Technology, and others. The utility of this compound depends on correct heat treatment, for unless the intermetallic compound is truly homogeneous, its thermoelectric power is low. Another compound of interest is lead sulfide, whose electrical properties are extremely sensitive to a slight excess of one or the other components.

Development of thermoelectric generators using such compounds has been retarded for years by the inability to make sufficiently good electrical connection to ingots of the material. Samples are cast from a melt, and are moderately brittle, as well as being subject to chemical attack. Many attempts at soldering resulted in failure to wet the surface on the one hand, or destruction of the ingot material (or at least its useful properties) on the other.

A recent development at The Franklin Institute Laboratories has provided one solution of the problem of making a practical thermoelectric generator.* Ingots of zinc-antimony are joined to a constantan strip by a combination of metal spraying and annealing.**

An ingot having a diameter of approximately half an inch is inserted in a hole of the same size punched in the strip. One side is then cleaned by sand blasting, and sprayed with suitable metal in such a way that the spray comes down cold and produces no oxidation. The joint is still mechanically weak, but an annealing process causes diffusion of the sprayed metal and good bonding, so that as annealing proceeds, mechanical strength increases and electrical resistance decreases.

Further researches will undoubtedly give more information about the process. Constantan spray works well, and likewise copper, though copper will oxidize and will combine with the constantan at elevated temperatures. Silver, while an excellent conductor, combines with the zinc and ruins the joint and eventually the ingot. Preliminary trials with aluminum show good bonding to the constantan but none to the ingot. Apparently the sprayed metal should have some affinity for the base, but not too much.

Joints so constructed have good mechanical strength, and high enough thermoelectric power combined with low enough electrical resistance to be suitable for practical application. If lead sulfide or other suitable material is developed having a high thermoelectric power, preferably of opposite sign to that of zinc-antimony, and if the new bonding technique works as well for other materials as it appears to do for zinc-antimony, then a practical, useful thermoelectric generator awaits only the design of an efficient heat-transfer system.

C. T. CHASE

* Work performed under contract with the Signal Corps Engineering Laboratories, Fort Monmouth, N. J.

** Author's note: A patent disclosure on the bonding technique has been made in the names of M. D. Earle, M. J. Disckind, and C. T. Chase.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

A NEW KIND OF PROPAGATION AT VERY HIGH FREQUENCIES

A new type of VHF propagation by means of the ionosphere, observable at distances as great as 1200 or 1300 miles from the transmitter, has recently been discovered. The investigation was cooperatively conducted by the National Bureau of Standards, the Massachusetts Institute of Technology, and the Collins Radio Co.[1] Early results reveal the uninterrupted presence of weak but observable signals over a 774 mile test path, and suggest that the signals are returned from a part of the E region of the ionosphere (between 50 and 70 miles above the surface of the earth) near or just below the absorption region for high-frequency radio waves.

The propagation of VHF radio waves (by definition, frequencies between 30 and 300 Mc.) to distances substantially beyond the horizon has been observed for many years. The propagation can nearly always be attributed to one of two causes. At short and intermediate ranges, special conditions prevailing at times in the troposphere usually provide an explanation. At greater distances, propagation is usually attributable to ionospheric reflections. Very high values of the F2-layer critical frequency (above which frequency the radio waves are not propagated) account satisfactorily for occasional cases of fairly strong fields at distances of several thousand miles, while sporadic E-layer ionization often accounts for erratic but strong fields out to distances of about 1200 miles.

It is known that the distribution of ionization in the ionosphere has a fine structure capable of producing incoherent scattering of radio waves. Even under quiet ionospheric conditions, there are turbulent motions in the ionosphere which lead to fading of single "rays" quite apart from fading due to interference between two or more rays. The question arose as to the possibility of detecting scattering arising from irregularities in the ionosphere when other forms of ionospheric transmission are absent. A theory on tropospheric scattering had been presented earlier by Booker and Gordon.[2] The need for experimental work was evident, and on January 23, 1951, suitable transmissions were initiated.

* Communicated by the Director.

[1] "New Kind of Radio Propagation at Very High Frequencies Observable over Long Distances," by D. K. Bailey, R. Bateman, L. V. Berkner, H. G. Booker, G. F. Montgomery, E. M. Purcell, W. W. Salisbury, and J. B. Wiesner, *Phys. Rev.*, Vol. 86, No. 2, p. 141 (April 15, 1952).

[2] "A Theory of Radio Scattering in the Troposphere," by H. G. Booker and W. E. Gordon, *Proc. I. R. E.*, Vol. 38, No. 4, p. 401 (1950).

The transmitter, operated by the Collins Radio Company at Cedar Rapids, Iowa, radiated continuous-wave signals at a frequency of 49.8 Mc., which were directed toward the National Bureau of Standards receivers at Sterling, Virginia at a distance of 774 miles. The RF power to a horizontal rhombic antenna was an estimated 23 kw. Similar antennas were used at both the receiving and transmitting locations, each 41.2 ft. high having 500-ft. legs and a tilt angle of 83°. Signals were received continuously from the outset, and the tests were continued in order to establish the diurnal and seasonal characteristics of the received signals and to establish the behavior of the signals during ionospheric disturbances.

A striking characteristic of the received signal, which exhibits fading characteristics that seem to follow a Rayleigh distribution, is the presence at all times of day of random heterodyne whistles, mostly descending in pitch. These are caused by reflections from the moving head of ionization accompanying meteors in their passage through the E region and the lower parts of the atmosphere. The whistles, which vary enormously in intensity according to the geometry of the situation and the size and velocity of the meteor, are relatively infrequent and of long duration in the evening, and frequent and of short duration in the dawn hours. Furthermore, a meteor giving a strong whistle sometimes causes the signal level to rise abruptly and to remain high for periods ranging from several seconds to as much as a minute or more. It seems likely that this meteoric residue makes an important contribution to the mean signal level.

A pronounced evening minimum of signal intensity is always observed. It is followed by a fairly steady rise during the night, a secondary maximum in the forenoon, and a midday maximum. Such diurnal behavior suggests that the received signal can be thought of as resulting from at least two distinct causes. One is the ionization produced by meteors, which make a maximum contribution at dawn and a minimum contribution at sunset. The other is the solar ultraviolet radiation, which peaks at midday and is absent at night. A superposition of the diurnal variations associated with these two causes, with relative importance varying according to season, would account reasonably for several features of the observed monthly diurnal characteristics.

The records of the received signals were carefully examined during times of magnetic disturbances and HF radio fadeouts. During the most severe magnetic disturbance for which observations were available, the signal showed pronounced intensification. During more moderate disturbances, the signal occasionally showed some enhancement, or appeared normal. Analyses of HF records during April and May 1951, showed 24 SID's (sudden ionosphere disturbance, a high frequency fadeout) occurring between 1½ hr. after sunrise and 1½ hr. before sunset for which simultaneous VHF data are available on the experimental path.

In 17 of these events, the received signal showed an increase ranging from 3 to 9 db during the period of the SID. This enhancement became noticeable within a few minutes of the onset of the SID, and the received signal returned to normal at the end of the fadeout. In the remaining seven cases of HF fadeout, the records of received VHF signal did not show any strengthening but appeared to be normal. An important point is that there was no evidence of any weakening of the signal for these cases.

These results are a strong indication that the received signal is returned from a part of the E region below the level of maximum ionization, and perhaps from below the absorbing region, sometimes referred to as the D region. This is further substantiated by earlier and independent observations of cosmic noise, which suffers a small but noticeable attenuation at the same frequency during SID's.

Other observations of the experimental transmissions have been made, notably by the Bell Telephone Laboratories, by Cornell University, and by radio amateurs. It is evident from these observations that the signal can be received over a considerable area. During a short period, receiving tests were also made in Bermuda, a distance of 1600 miles from Cedar Rapids. Signals were received sporadically, and only at times associated with intense sporadic E ionization at some point along the path.

In the pre-experimental calculations, it was estimated that the received signal power corresponding to a critcial frequency of 1.55 Mc. and coming from a height of about 100 kilometers would be 2.0×10^{-12} watt. Inspection of records revealed that the maximum median values of received signal varied in the middle of the day from power levels of about 4.2×10^{-15} to 1.7×10^{-13} watt. These values, which represent very weak signals, occurred at times when the highest usable frequency for the usual kind of E-layer reflection for this path varied from 13.8 to 14.4 Mc.—well below the operating frequency of 49.8 Mc. It is thus apparent that the pre-experimental speculations were too optimistic. The behavior of the signal during SID's suggests that the ionospheric region responsible for the transmission may be lower than 100 kilometers. At such lower heights the values of the critical frequencies corresponding to the electron densities normally present are substantially lower, although probably exhibiting similar diurnal and seasonal variations. It would thus appear that the adaptation of the Booker-Borden theory to the ionosphere could better account for the experimental results if values of critical frequency were used which are appropriate to the lower heights from which the signals seem to come.

Results of experimentation to date, while of considerable interest, leave many questions unanswered. For future experimentation, tests of frequency and distance dependence are contemplated, as well as more detailed study of the nature of the received signal, with the objective of

improving understanding of the transmission mechanism and the parameters of the medium.

TALKING BOOKS FOR THE BLIND

An investigation of the many problems involved in recording and reproducing human speech is being conducted by the National Bureau of Standards. The present program is the outgrowth of assistance which the Bureau rendered the Library of Congress in improving the quality of "talking books"—voice recordings of selected literature which the Library supplies to the blind. In addition to performance studies of recording and reproducing equipment, new and improved methods of test were worked out, and new types of recording media were investigated. In the course of this work, a simple yet highly effective technique for visual analysis of speech sounds was developed.

In talking books high fidelity is especially important because most of the material is unfamiliar to the listener, and he has no easy way of checking his impression of the words he hears against the printed words in the book. Because of this, talking-book recordings must have higher fidelity than is usual even in good musical recordings. Special, high-quality records and reproducing equipment are thus required. As the Library has invested several million dollars in talking books, it requested the Bureau to investigate talking-book production techniques in current use and to set up specifications for the purchase of records and reproducers.

The program began with the preparation of an interim specification for talking-book phonographs. When sample reproducers were tested for compliance with this specification, large variations were found in such charateristics as tip radius of the reproducer needle, tone-arm force, and loudspeaker response. It was also evident that standardization of record characteristics was needed; arrangements were therefore made with two of the largest suppliers of talking-book records for a cooperative study of their recording techniques, with the ultimate object of developing a standard technique.

Since there were no established methods for testing speech records, it was necessary for the Bureau to devise a new technique for studying speech communication systems. This technique developed by Edith L. R. Corliss and associates of the Bureau staff, utilizes a time-integrated spectrogram of speech sounds which can be interpreted to show many of the characteristics of the transmitting system. Although single speech sounds, or syllables, are complex patterns of different frequencies and sound levels, over a relatively short period of time nearly all sound levels and frequencies within the entire speech spectrum are produced. The characteristics of a recording system can thus be determined by comparing an integrated spectrogram of the human voice with an integrated spectrogram for the recording system.

To provide the necessary sound spectrograms, a commercial spectrograph known as a Panoramic Analyzer was modified in the Bureau's sound laboratory. Ordinarily the Panoramic Analyzer sweeps through the audio-frequency range once a second and displays a spectrum of the signal at its input on the face of a cathode-ray tube. The horizontal coordinate of the pattern is a frequency scale; the vertical display is proportional to the amplitude of each frequency contained in the input signal. Because speech sounds change in composition more often than once a second, many of the frequencies present in a sample of speech would not be included in a one-second sweep. It was therefore necessary to modify the Panoramic Analyzer so that it would sweep through its analysis range at the syllable articulation rate. This made it possible to display instantaneous spectrograms of individual syllables on the cathode-ray screen when the equipment was connected to the output of a microphone. By exposing photographic film for sufficient time (at least 60 seconds) to successive syllable spectrograms, an integrated envelope characteristic of the speech communication system under study is obtained.

A simple optical procedure, previously developed by Buchmann and Meyer in Germany, is used to relate these integrated spectra to the actual characteristics of the talking book disks as manufactured. In this procedure, the pattern formed by reflection of a collimated beam of light from the groove faces of a record—made up of a series of single-frequency bands—is measured and used to calibrate the modulation of the record directly, eliminating the effect of the reproducer system. After a test record has been calibrated in this way, the integrated frequency spectrograms of the talking books under study are compared with the spectrogram of the calibrated test record.

The spectrographic method developed by NBS offers a powerful tool for the study not only of speech communication systems but of the nature of speech itself. Previous methods for visually observing speech characteristics either required the repeated playing back of a recording in order to make a frequency analysis, or else sacrified a certain amount of resolving power by using fixed-frequency band filters and multiple channels to obtain the necessary high speed. The Bureau's method, on the other hand, allows direct observation of the location of the peak frequencies making up speech sounds and of their relative magnitudes with a resolving power equivalent to that of the fixed band filters, but it operates through a single channel. Thus, the modifications in the structure of the speech sounds produced by either the recording process or the reproducer system may be determined with some precision. In addition, the statistical occurrence of speech sounds and their inherent variability can be studied by this means.

The Bureau hopes to make further studies of the new method for analyzing speech communication systems. Interesting subjects for in-

vestigation include the statistical distribution of sound levels and frequencies in speech, the amount of time needed for obtaining representative samples of speech recordings, and the effects which such properties as frequency-range restriction, noise, resonances, and harmonic distortion have on the photographed pattern.

On a disk recording the high-frequency modulation is very small in size. As a result, there is a tendency for the intensities of some of the higher frequencies to be reduced, making certain of the consonant sounds less easily distinguished. To provide data for improved specifications for talking book records, studies have been carried out to determine the optimum pre-emphasis of high frequencies during the recording process and to ascertain what change of pre-emphasis with recording radius is necessary to maintain high-frequency fidelity in the inner radii of talking-book disks.

Recently, the Bureau made a study of the durability of various types of luggage used for reproducer housings. This information, needed in connection with the drafting of a new specification for the talking-book reproducers, should result in considerable savings in repair costs to the Library. Both the durability of the housing itself and the protection afforded to its contents were investigated. Mechanical tests which simulated the treatment the housings would be expected to receive in the mail were carried out in the NBS leather laboratory. Several possible mountings for the loudspeaker which might reduce distortion and mechanical feedback were also devised and are under test.

At the request of the Library, the Bureau has been investigating the possibilities and limitations inherent in recordings produced by embossing processes such as are used in the temporary recording of dictation for transcription. A large number of text books have been recorded on embossed disks for blind students by volunteer workers, and it would be of advantage to talking-book users if these recordings could be made more generally available. In addition, the lightness and small size of embossed disks would be advantageous for the talking-book system because of the necessity of circulating the records through the mail. However, the plasticity of the embossed disks, the fineness of their grooves, and the limited range of sound frequencies that can be recorded on them are difficulties that must be overcome if they are to be useful to blind readers as a reasonably enduring library. At the present time the highest frequency that can be recorded satisfactorily on embossed disks is too low to permit clear recognition of "s," "th," "b," and "v" sounds in unfamiliar words. Although this limitation is not a great disadvantage in office dictation, where a familiar vocabulary is usually employed, the high-frequency limit of the embossed recording is a handicap to the blind reader seeking to acquire new information.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

***Abstract of* An Improved Colorimetric Method for the Quantitative Estimation of Thymine.**—E. D. DAY AND W. A. MOSHER (*J. Biol. Chem.*, 197, 227 (1952)). Thymine, a constituentof desoxypentosenucleic acids (DNA), had been found by Hunter to produce an intense red color when coupled with *p*-diazobenzenesulfonic acid. It had been measured in hydrolysates of DNA according to the Hunter procedure by Woodhouse, by Pircio and Cerecedo, and by Day *et al.* The Hunter procedure however has been found to be limited to semiquantitative determinations of thymine. An improved method was therefore developed for the quantitative estimation of thymine in hydrolysates of DNA.

The coupling reaction for thymine was carried out under pure oxygen; the diazo reagent of Koessler and Hanke was employed at twice the concentration used in previous procedures; a mixture of glycerol and sodium hydroxide was substituted for hydroxylamine to give an intense and stable red color; the values of optical density of colors produced with cytosine and uracil were nullified by appropriate blanks when these pyrimidines appeared as contaminants in thymine solutions; and standard equations relating thymine concentrations and optical densities were deduced for the quantitative estimation of thymine in solutions of pure thymine and in solutions of thymine contaminated with cytosine and uracil.

The isolation of the pyrimidine fraction from the sulfuric acid hydrolysate of DNA was accomplished by a simple procedure which permitted the subsequent colorimetric estimation in this fraction of thymine.

THE FRANKLIN INSTITUTE

MEDAL DAY

Wednesday, October 15, 1952

The annual Medal Day ceremonies of The Franklin Institute were held on Wednesday, October 15, 1952, in the Benjamin Franklin Memorial Hall. Approximately 360 persons attended the dinner and the presentation of the awards. Prior to the dinner, a reception in honor of the 1952 Medalists was held in the rooms adjoining Franklin Hall.

The ceremonies opened with the playing of the National Anthem by Mr. Guy Marriner, Director of Music at The Franklin Institute. The invocation was said by the Reverend R. Dunham Taylor, Rector of the Memorial Church of the Good Shepherd, Germantown.

The Stated Monthly Meeting of The Franklin Institute was held in conjunction with the Medal Day exercises. Mr. S. Wyman Rolph, President, presided. The Minutes of the Stated Meeting in May were approved as published in the June issue of the JOURNAL. Mr. Rolph announced that Dr. Wolfgang Pauli, recipient of the Franklin Medal, had been elected to honorary membership in the Institute and that his certificate of membership would be presented to him *in absentia*.

Mr. Howard Stoertz, Chairman of the Committee on Science and the Arts, then introduced the thirty-three former Medalists who were present, and also the thirteen former Company Medalists who were represented.

Dr. Henry B. Allen, Executive Vice-President and Secretary, was called upon to give his report on "The Institute in 1952."

Following Dr. Allen's report, the presentation ceremonies for the twelve medals and one Certificate of Merit awarded in 1952 were opened by a short message from the President. The sponsor for each Medalist read the citation accompanying the award and introduced the Medalist. In the absence of Dr. Wolfgang Pauli, of Zurich, Switzerland, whose duties made it impossible for him to be present, Mr. Maurice J. Rohrbach, Consul for Switzerland in Philadelphia, stood as proxy.

At the conclusion of the presentation of the awards, Mr. Rolph introduced Dr. I. Bernard Cohen (Department of the History of Science and Managing Editor of *Isis*, Harvard University), who spoke most interestingly on "Franklin and His Lightning Rod."

The meeting adjourned at 10:00 p.m.

The full proceedings of Medal Day will be published in the December issue of the JOURNAL.

HENRY B. ALLEN,
Secretary

PROGRAM

Reception for Medalists	THE HOSTESS COMMITTEE
National Anthem	GUY MARRINER
Invocation	THE REVEREND R. DUNHAM TAYLOR
Dinner	MEDALISTS, OFFICIALS AND GUESTS
Toast to Benjamin Franklin	S. WYMAN ROLPH, *President*
Greetings	THE PRESIDENT
Stated Meeting of the Institute	THE PRESIDENT PRESIDING
Introduction of Former Medalists	THE PRESIDENT
The Institute in 1952	HENRY B. ALLEN, *Executive Vice-President*
Presentation of Awards	THE PRESIDENT
"Franklin and His Lightning Rod"	I. BERNARD COHEN

MEDALISTS

To be Awarded a Certificate of Merit (1882)

ARTHUR M. STONER

Vice-President
The Jacobs Manufacturing Company
West Hartford, Connecticut

To be Awarded Cresson Medals (1848)

EDWARD C. MOLINA

Telephone Engineer
East Orange, New Jersey

H. BIRCHARD TAYLOR

Engineer, Industrialist and Author
Haverford, Pennsylvania

To be Awarded a Henderson Medal (1924)

ALFRED J. BUCHI

Engineer
Winterthur, Switzerland

To be Awarded Wetherill Medals (1925)

HARRISON P. HOOD

Research Chemist
Corning Glass Works
Corning, New York

MARTIN E. NORDBERG

Research Chemist
Corning Glass Works
Corning, New York

ALBERT J. WILLIAMS, JR.

Associate Director of Research
Leeds & Northrup Company
Philadelphia, Pennsylvania

To be Awarded a Clark Medal (1926)

EDWIN L. HALL

Director, Laboratories
American Gas Association, Inc.
Cleveland, Ohio

To be Awarded a Brown Medal (1938)

FRED N. SEVERUD

Consulting Engineer
New York, New York

To be Awarded a Clamer Medal (1943)

CYRIL STANLEY SMITH

Professor of Metallurgy, and Director of the Institute for the Study of Metals
The University of Chicago
Chicago, Illinois

To be Awarded Ballantine Medals (1946)

JOHN BARDEEN

Professor of Physics and Electrical Engineering
University of Illinois
Urbana, Illinois

WALTER H. BRATTAIN

Research Physicist
Bell Telephone Laboratories, Inc.
Murray Hill, New Jersey

To be Awarded the Franklin Medal (1914)

WOLFGANG PAULI

Physikalisches Institut Polytechnicum
Zurich, Switzerland

MUSEUM

The gift of the steering wheel of U. S. Monitor *Cheyenne* to the Museum serves to remind us that the problem of steering a ship has not always been an easy one. The late Professor Charles M. Gay, who was largely responsible for the creation of the Marine Transportation Section, realized this and made the excellent series of models which traces the history of steering devices.

It is difficult for the modern mind to realize that the clumsy early ships were steered without the aid of a rudder. On the Mediterrean, where the ship had its origin, steering was performed with heavy oars. For the larger sea-going vessels, two oars were used, one on either side of the ship. The independent development of the ship which took place in the North of Europe saw the introduction of a single oar on the swift light Viking ships. This oar was located on the right, or starboard quarter (*stjonbordi*-starboard or steering side) and it persisted until the rudder was invented.

The stern rudder appears to have developed independently in China and Northwestern Europe. We get a probable date for the introduction into the latter area about the 12th century from the pictures which appear on the city seals of seaports. At this period the ship was becoming much heavier and, consequently much more difficult to row with oars. This led to the introduction of more masts to carry sails, and to the cultivation of an ability to sail to windward, which had not been possible. When the ship heeled over in this maneuver under the pressure of a strong breeze on the starboard bow, the steering paddle would be raised out of the water. Once a centrally situated steering paddle was applied, its evolution into the rudder, controlled by a tiller, was relatively quick and widespread.

The introduction of the rudder had the effect of changing the shape of ship which, formerly had been the same at the bow and the stern. To permit the rudder to be fitted to the sternpost and fitted with a pivot near the lower end, the stern of the ship had to be rounded and flattened.

Since the tiller was on the spar deck, the helmsman had no view of his course and changed the rudder on orders called down a hatchway by the officer on deck. As the ships grew in tonnage and had to have correspondingly heavy rudders, it was not always possible for one man to handle the tiller, especially in heavy seas. This difficulty, together with a growing mechanical sense, led to the introduction of the steering wheel about the turn of the 18th century. This was originally operated by ropes passed around a drum.

The old steering wheels were quite impressive in their size. The specimen shown through the generosity of Mr. R. C. Morse, of Radnor, Pa., was taken from the *Cheyenne*, formerly the *Wyoming*, 3225 tons, and built in 1899. She was struck off the Naval Register in 1937. The visitor is recommended to visit the ship's bridge and to note how the wheel has been reduced in size and, incidentally, to observe how the old terms port and starboard are passing from the nautical vocabulary.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

SHEN, D. W. C. AND H. N. G. BROADBENT: Analysis of Partly Symmetrical Machines by Means of Unitary Transformation.

GROSS, ERIC T. B.: Unbalances of Untransposed Overhead Lines.

SAIBEL, EDWARD AND WINSTON F. Z. LEE: Vibrations of a Continuous Beam Under a Constant Moving Force.

URKOWITZ, HARRY: A Ferroelectric Amplifier.

SILBERSTEIN, LUDWIK: Elementary Probability of Efficient Absorption of a Quantum Proportional to the Volume of a Grain.

KU, Y. H.: Nonlinear Analysis of Electro-Mechanical Problems.

ARSOVE, MAYNARD G.: The Algebraic Theory of Linear Transmission Networks.

MOON, PARRY AND DOMINA EBERLE SPENCER: Theory of the Photic Field.

MOON, PARRY AND DOMINA EBERLE SPENCER: Some Applications of Photic Field Theory.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

RECENT ADDITIONS

AGRICULTURE

WILLIAMS, W. R. Principles of Agriculture. 1952.

BIOLOGICAL CHEMISTRY

CIBA FOUNDATION. Conference on Isotopes in Biochemistry. 1951.

CHEMISTRY AND CHEMICAL TECHNOLOGY

BLOCK, RICHARD JOSEPH; LESTRANGE, RAYMOND AND ZWEIG, GUNTER. Paper Chromatography. 1952.

COWIE, G. A. Potash. 1951.

DOSS, MILBURN PRICE. Properties of the Principal Fats, Fatty Oils, Waxes, Fatty Acids and Their Salts. 1952.

DREISBACH, ROBERT RICHERT. Pressure-Volume-Temperature Relationship of Organic Compounds. Ed. 3. 1952.

DU VIGNEAUD, VINCENT. A Trail of Research in Sulfur Chemistry and Metabolism and Related Fields. 1952.

EITEL, WILHELM. Thermochemical Methods in Silicate Investigation. 1952.

FRIEND, JOHN ALBERT NEWTON. Man and the Chemical Elements from Stone-age Hearth to the Cyclotron. 1951.

FRIEDRICHS, FRITZ. Das Glas im Chemischen Laboratorium. 1951.

GUGGENHEIM, EDWARD ARMAND. Mixtures. 1952.

HECHT, HORSTMAR. Präparative Anorganische Chemie.

HOFMANN, KARL ANDREAS. Anorganische Chemie. Ed. 14. 1951.

HUNTER, FRANCIS T. The Quantitation of Mixtures of Hemoglobin Derivatives by Photoelectric Spectrophotometry. 1951.

JAKOB, J. Chemische Analyse der Gesteine un Silikatischen Mineralien. 1952.

KLEMENC, ALFONS. Anorganische Chemie auf Physikalisch-Chemischer Grundlage. 1951.

KÜCHLER, LEOPOLD. Polymerisationskinetik. 1951.

MARTELL, ARTHUR EARL AND CALVIN, MELVIN. Chemistry of the Metal Chelate Compounds. 1952.

MITTASCH, ALWIN. Geschichte der Ammoniaksynthese. 1951.

RODD, E. H., ED. Chemistry of Carbon Compounds. Vol. 1. 1952.

RUMFORD, FRANK. Chemical Engineering Operations. 1952.

RUMMEL, THEODOR. Hochspannungsentladungschemie. 1951.

STUART, H. A. Die Struktur des Freien Moleküls. 1952.

VAIL, JAMES GARRETT. Soluble Silicates. Vol. 1. 1952.

WAESER, BRUNO. Chemisch-technische Arbeitsgänge und Apparateuren. 1951.

WINKLER, HENRI JOSEPH. Der Steinkohlenteer und seine Aufarbeitung. 1951.

WIBERG, EGON. Die Chemische Affinität. 1951.

CRYSTALLOGRAPHY

HALLA, FRANZ. Kristallchemie und Kristallphysik Metallischer Werkstoffe. Ed. 2. 1951.

NOWACKI, WERNER. Fouriersynthese von Kristallen und ihre Anwendung in der Chemie. 1952.

DICTIONARIES

Handy Technical Dictionary in 8 Languages. 1949.

ECONOMICS

BRITTAIN, ROBERT. Let There be Bread. 1952.

ELECTRICITY AND ELECTRICAL ENGINEERING

ANDERSON, EDWIN P. Audels Electronic Devices With Industrial Applications. 1952.
BISHOP, CALVIN C. Electrical Drafting and Design. Ed. 3. 1952.
Conferences on Automatic Control. 1952.
ERICKSON, WILLIAM H. AND BRYANT, NELSON H. Electrical Engineering. 1952.
SCHUISKY, W. Elektromotoren ihre Eigenschaften und ihre Verwendung für Antriebe. 1951.
SOMMERFELD, ARNOLD. Electrodynamics. 1952.

ENGINEERING

DEN HARTOG, JACOB PETER. Advanced Strength of Materials. 1952.
MCCORMICK, FRANK J. Strength of Materials. 1952.

ELECTRONICS

MANLY, HAROLD PHILLIPS. Television Servicing for Radiomen. 1949.

FOOD

INSTITUTE OF BIOLOGY. Freezing and Drying. 1952.
LODDER, J. AND RIJ, N. J. W. The Yeasts. 1952.

GENERAL

PARSONS, JOHN E. Henry Deringer's Pocket Pistols. 1952.

HISTORY

BRODERICK, ALAN HOUGHTON. The Tree of Human History. 1951.

MANUFACTURE

MOAKES, R. C. W. AND WAKE, W. C. Rubber Technology. 1951.
MOSHER, ROBERT H., ED. The Technology of Coated and Processed Papers. 1952.
SIEBER, RUDOLF. Die Chemisch-technischen Untersuchungs Methoden der Zellstoff und Papier Industrie. Ed. 2. 1951.

MATHEMATICS

ASMUS, E. Einführung in die Höhere Mathematik. Ed. 2. 1952.
CUCCIA, C. LOUIS. Harmonics, Sidebands and Transients in Communication Engineering as Studied by the Fourier and Laplace Analyses. 1952.
HILBERT, DAVID AND COHN-VOSSEN, S. Geometry and the Imagination. 1952.
SALMANG, HERMANN. Die Physikalischen und Chemischen Grundlagen der Keramik. Ed. 2. 1951.
SEMPLE, JOHN GREENLEES AND KNEEBONE, G. I. Algebraic Projective Geometry. 1952.
SPEAR, MARY ELEANOR. Charting Statistics. 1952.
WAGNER, KARL WILLY. Operatorenrechnung und Laplacesche Transformation. Ed. 2. 1950.

MECHANICAL ENGINEERING

DUSINBERRE, G. M. Gas Turbine Power. 1952.
ST. CLAIR, LEO J. Design and Use of Cutting Tools. 1952.

METALLURGY

HOUGHTON, E. F., & CO. Practical Metallurgy for Engineers. Ed. 5. 1952.
SCHUHMANN, REINHARDT, JR. Metallurgical Engineering. Vol. 1. 1952.
SILAMN, H. Chemical and Electroplated Finishes. Ed. 2. 1952.
U. S. BUREAU OF STANDARDS. Mechanical Properties of Metals at Low Temperatures. 1951.

PHARMACY AND PUBLIC HEALTH

HINSBERG, KARL AND MERTEN, RICHARD. Chemische Bestimmungs Methoden im Klinischen Laboratorium. 1952.

WINTERFELD, KARL. Praktikum der Organisch Präparativen Pharmazeutischen Chemie. Ed. 3. 1950.

PHYSICS

BLOCH, S. S. Angenäherte Synthese von Mechanismen. 1951.

DENNE, W. Magnetic Compass Deviation and Correction. 1951.

FEININGER, ANDREAS. Advanced Photography. 1952.

JAGGER, J. G. A Textbook of Mechanics. 1952.

MASSEY, HARRIE STEWART WILSON AND BURHOP, E. H. S. Electronic and Ionic Impact Phenomena. 1952.

ROYDS, ROBERT. The Measurement and Control of Temperature in Industry. 1951.

SIMON, F. E. AND OTHERS. Low Temperature Physics. 1952.

SÖCHTING, FRITZ. Berechnung Mechanischer Schwingungen. 1951.

PLASTICS

BOURRY, J. Résines Alkydes-Polyesters. 1952.

KAINER, FRANZ. Polyvinylchlorid und Vinylchlorid-Mischpolymerisate. 1951.

SCIENCE

JOHNSON, MARTIN. Science and the Meaning of Truth. 1952.

MELSEN, ANDREW G. VAN.* From Atomos to Atom. 1952.

TEXTILES

LÜTTGEN, C. Organische und Anorganische Wash-, Bleich-, und Reiningsmittel. 1952.

MONCRIEFF, R. W. Mothproofing. 1950.

OESER, WERNER. Baumwoll und Zellwollspinnerei. 1950.

TRANSPORTATION

CZITARY, EUGEN. Seilschwebebahnen. 1951.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, OCTOBER 15, 1952

ACTIVE FAMILY

John J. Entwistle
Charles O. Marron
Philip G. Schmaelzle
Chester C. Williams

ACTIVE

Daniel Amoroso
Vernon F. Bailie, D.D.S.
William S. Campbell
Capt. William B. Chatfield
F. A. Fendel, Jr.
Edward L. Fortin
Benjamin Fox
Alan D. Franklin, Ph.D.
Edward S. Gifford, Jr., M.D.
Edward P. Goehring
William S. Graff
Bernard Hantower
Walter E. Krause
Walter M. Kulzer
Joseph Parrish
Stanley N. Prickitt
Tommy J. Prugh
Ralph H. Randall
Isadore Rosenthal
Meyer Samson
James J. Schattler
Charles E. Scheidig
Thomas D. Scholl
Wilfred F. Skeats
Amos C. Smethers
Roland M. Summers
Harry P. Swing
Norman Virkler
Frank H. Walker

ACTIVE NON-RESIDENT

John A. Dickinson
William H. Parmelee

NECROLOGY

William J. Crout '42
Richard C. McCall '16
James J. Pocock '36
Joseph M. Pugh '44
Miss Florence Sibley '47
Frank H. Thomas '44

BOOK REVIEWS

MECHANICS. PART I: STATICS, by J. L. Meriam. 340 pages, diagrams, 15×23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $4.00.

MECHANICS. PART II: DYNAMICS, by J. L. Meriam. 331 pages, diagrams, 15×23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $4.00.

In reviewing a book on mechanics, the oldest of the physical sciences and extending back to the time of Archimedes, there is little need to discuss the material included, since it naturally varies but slightly from any other text. For instance, a book on mechanics, usually divided into statics and dynamics, would be expected to contain, under the first part, sections on force systems, equilibrium, structures, beams and friction; and, under the second part, to contain something on kinematics, kinetics, work and energy, impulse and momentum, and vibration. Such indeed are the contents of this recent set of books on "Mechanics" by Prof. J. L. Meriam of the University of California. Since there is nothing new in the material itself, there must be other reasons for writing and publishing a book on such an old subject; perhaps the coherence of presentation and the style of writing are more clear; perhaps the connection between the physical and the theoretical is more readily explained; perhaps the illustrations are better; even, perhaps, the printing and format of the book itself are better. And, no doubt, there are other reasons.

Reading this book, one quickly realizes that there are indeed reasons for publishing it. In addition to its being generally well written and the material being clearly and logically presented and developed, one is immediately impressed, in fact almost overwhelmed, with the number and variety of problems contained in the two volumes on mechanics. It is Prof. Meriam's firm belief that the sure way to make a student understand mechanics—and there is none better—is to show him the connection between theory and practice; that is, the application of the theory to practical engineering problems. This is the essence of his method: a new subject is explained part by part, each part followed by several illustrative problems, and then by a number of original problems demonstrating the commonplace applications of the theory to be solved by the student. Although the method is far from novel, it has seldom been executed with such thoroughness. The more than 1200 problems, many accompanied by clear, excellent figures, ranging from exercises in simple trusses, beams and structures, through mechanical gear trains, various complicated machine design parts, and stability of boats, to jet propulsion and the vibration of eccentric disks will give anyone a broad background in the application of mechanics to engineering, as well as an appreciation of the important part mechanics plays in engineering.

It should also be noted that a book presented in such a way can be studied and readily understood by one working alone without benefit of teacher.

E. W. HAMMER, JR.

CORROSION TESTING PROCEDURES, by F. A. Champion. 369 pages, diagrams, 15×22 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.25.

Many books have been written on corrosion, and the economic importance of this subject is well recognized. The factors which influence the corrosion of metals are so numerous that it is seldom possible to bring them all under control economically. Fundamental to a knowledge of corrosion characteristics of metals is an understanding of the means for measuring these properties. In fact, this is the key to an accurate appraisal of corrosion-resistance characteristics which are widely tabulated.

The objective of corrosion testing is to perform experiments which will predict behavior of a metal under the conditions of its use. Since the latter may vary widely, one attempts to make these experiments as fundamental as possible. That is, it would be desirable to reduce the entire subject to simple chemical formulae. This being impossible, attention shifts to

ways of standardizing the testing procedures in the hope that they will bear a close relation to the actual conditions causing corrosion.

When one considers the great variety of ferrous and non-ferrous alloys, the magnitude of the subject becomes apparent. The breadth of the problem is increased to barely manageable proportions when the variety of factors and possible combinations of factors producing corrosion is considered. The author, F. A. Champion, has spent a great deal of time studying these problems and has presented in his book various techniques which result in an organized procedure for testing metals.

Before considering any of the laboratory tests, certain basic problems are encountered in which it is necessary to standardize the preparation of the material to be tested as well as methods for assessing the effects of the treatment. Furthermore, one is faced squarely with the problem of separating conditions that influence corrosion activity so that a great jumble of meaningless data is not produced.

For those actually performing the tests, the book describes specific arrangements of chemical apparatus and the necessary reagents for laboratory exposure. Numerous mechanical contrivances for producing intermittent immersion, salt spray, controlled atmospheres, mechanical stresses and many other conditions are illustrated.

For the assessment of corrosion effects a great many techniques are described which measure the change or loss of mechanical properties, electrical characteristics and appearance. Various ancillary observations dealing with potential and conductivity measurements are of interest. Electronic circuits useful in making these measurements are described in some detail. Mention is made of artificially produced thin films and the use of a polarizing spectrometer in assessing their thickness on metals. The electrometric method for determination of film thicknesses by reduction methods is explained, and practical examples of its use are described.

The book should be valued by persons working in industrial testing laboratories and with metals generally. It also provides much useful information of a general engineering nature. Designers as well as those responsible for type-testing of equipment will find the book a practical reference.

C. W. HARGENS

ADVANCED STRENGTH OF MATERIALS, by J. P. Den Hartog. 379 pages, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $8.50.

Many books have already been written on the strength of materials and, offhand, another one on the same subject is hardly cause for excitment. However, a review of "Advanced Strength of Materials" indicates it is different from the usual text in several respects. First, it has been written to fill the gap between the elementary text on strength of materials and such advanced texts as those of Timoshenko's on plates and shells and elastic stability. Second, it has been written in the same easy, conversational style that characterizes Prof. Den Hartog's lectures, which means the material is clearly presented. And, third, the physical as well as the theoretical or mathematical treatment of the subject has been clearly explained. This last is particularly important for it makes the understanding much easier and the retention much surer. It is indeed a pleasure to have a book on such theoretical material in which the physical significance of the theory, in most cases, is so lucidly delineated. On these counts, then, "Advanced Strength of Materials" is definitely a notable addition to our technical literature.

In discussing the specific subjects covered, let us say at the outset, in order to save repetition, that generally each subject is treated with a clarity and thoroughness infrequently found in such texts. Such treatment indicates a sound comprehension of the material by the author. And when such is the case, a good book generally results. Professor Den Hartog begins with the subject of torsion in which are explained Saint-Venant's theory, Prandtl's membrane analogy, Kelvin's fluid-flow analogy and Jacobsen's electrical analogy. If one has experienced difficulty in understanding these analogies, he should read Prof. Den Hartog's discussions, for they are certainly made clear here.

The stresses in rotating and non-rotating disks of various shapes are next treated. Thorough discussions of the general theory, practical applications and unusual cases of stresses in shells, bending of flat plates and beams on elastic foundations follow. Under the two-dimensional theory of elasticity, Prof. Den Hartog shows the simplification in the theory made possible by the introduction of the Airy stress function and its application to several classical problems such as the stress concentration around a hole in a plate and the stress distribution in a plate due to a load on the edge. Several problems in plasticity, such as a thick cylinder under internal pressure, are also discussed.

While perhaps not as understandable as other portions of the book (which can very well be due to the peculiarities of the reviewer) the explanation of the three energy theorems—virtual work, least work and Castigliano's theorem—is unusually complete, including, as it does, theory, physical interpretation and practical application. Under buckling, column bending and twisting stability, beams on elastic foundation, rings and tubes and arches, the twist-bend buckling of beams, and the buckling of torsion shafts are described. In the concluding chapter are treated such miscellaneous topics as Mohr's circle for three dimensions, the torsion of pre-twisted thin-walled sections, and the determination of the end reactions of a steam pipe fixed at both ends subjected to a change of temperature by the theorem of Castigliano. More than a hundred excellent problems with answers complete the volume.

In addition to his extensive theoretical background, Prof. Den Hartog has had considerable practical experience. Throughout "Advanced Strength of Materials" numerous illustrative problems reflect this practical background. The solutions to such problems as the stresses in rotating turbine disks, in vertical oil or water tanks, in the dome of a building, and in the beams of a building floor grid; and the bending of long thin-walled tubes are examples of such every-day engineering applications to be found in this most recent and superior book on strength of materials.

E. W. HAMMER, JR.

LOW TEMPERATURE PHYSICS, by F. E. Simon, N. Kurti, J. F. Allen, and K. Mendelssohn. 132 pages, diagrams, 14×22 cm. New York, Academic Press, Inc., 1952. Price, $3.50.

Low temperature physics, one of the most fascinating fields of science, is today one of the most rapidly growing branches of physics. Inaugurated at the University of Leiden with the first liquifaction of helium by Kammerlingh Onnes in 1908, low temperature research is at present in progress in scores of university and research laboratories throughout the world.

This book, a series of lectures presented by four of the foremost authorities on the subject, is at once delightful and informative. Although its limited size imposes a sometimes regrettable restriction upon the details of the treatment, it provides an excellent summary of the topics which have been of major interest in low temperature research.

Professor Simon, in the first lecture, discusses in a general way some of the problems and possibilities of the subject. This is followed by a lecture by Dr. Kurti on the temperature range below 1 degree Absolute. Beginning with general considerations concerning the cooling of a substance, Kurti continues to discuss the method of adiabatic demagnetization of paramagnetic salts in use today, and finally considers a subject which has barely been touched experimentally, nuclear paramagnetism. Professor Allen next describes the remarkable properties of liquid helium, itself a topic which has long occupied many able investigators. Finally, Dr. Mendelssohn discusses the equally intriguing topic of superconductivity in metals. The third and fourth lectures, particularly, show that although much has been learned about low temperature phenomena, a thorough understanding of their many aspects requires still a great deal of further study.

Although not by any means an advanced treatise on its subject, this book should fulfill its stated purpose of providing an introduction to the field for the student and the scientist with an indirect interest in low temperature physics.

D. P. DETWILER

MORPHOGENESIS: AN ESSAY ON DEVELOPMENT, by John Tyler Bonner. 296 pages, illustrations, 15×22 cm. Princeton, Princeton University Press, 1952. Price, $5.00.

This book is just what it claims to be, an essay on development. Such subjects as patterns of growth, polarity, symmetry, differentiation, and morphogenetic development are

discussed. Emphasis is principally on the lower invertebrates and plants, although the author deals with some problems of higher forms. The sections dealing with the slime bacteria and amoeboid slime molds in the chapter on morphogenetic movement should interest anyone in the field of biology.

The author makes no attempt to explore exhaustively the various topics under consideration and, indeed, therein lies much of the merit of the book. The view presented is essentially a broad one and the style in which it is presented is a welcome change from usual scientific writing. One may not agree with some of the analogies or interpretations but there can be no doubt of the value of the book in stimulating thought upon this very important subject. The book is indeed an excellent essay.

WARREN R. STINEBRING

THE EXACT SCIENCES IN ANTIQUITY, by O. Neugebauer. 191 pages, illustrations, 16×24 cm. Princeton, Princeton University Press, 1952. Price, $5.00.

Civilization is composed of many facets and the attempt to trace its course through the ages must, of necessity, spread into many and varied fields. As an example, the mathematical attainments of a civilization are indicative of a wide knowledge of astronomy for these two sciences are inextricably linked. By following the connection of mathematics and astronomy the researcher can at times move from era to era—from civilization to civilization in an unbroken chain.

In "The Exact Sciences in Antiquity," Dr. Neugebauer, professor of the History of Science at Brown University, brings for the first time in book form the results of his most recent researches on Babylonian mathematics and the interrelations of Greek, Mesopotamian and Hindu scientific culture. He shows the Babylonians, a thousand years before Pythagoras, had not only an intimate knowledge of geometry but also possessed "astonishing facility" with quadratics and higher order equations. He shows a continuous tradition must have existed, linking Mesopotamian mathematics of the "Hellenistic" period with contemporary Semitic and Greek writers and finally with the Hindu and Islamic mathematicians.

The book is a thoroughly fascinating one, skillfully written, and destined to find its way into the libraries of not only students of antiquity but also mathematicians and astronomers. The notes and references at the conclusion of each chapter are, to this reviewer, as important as the lectures themselves.

There appear to be some disturbing elements to be found in the book and, if they are substantiated, many public astronomers will have to alter their presentation to conform to this new interpretation.

The author shows that the Babylonian system completely disregarded the arrangement of the planets in space. He maintains that it was the Greeks who followed this pattern which led to the seven day week and the naming of the days after the planet ruling the first hour of the day. The division of the day into 24 hours is a form of reckoning not Babylonian but a "Hellenistic" product of ultimately Egyptian origin. These assertions are contrary to the present day accepted ideas.

The division of the day into 24 hours and the week into seven days was a phenomenon which must have had a common origin. Otherwise how could the week have originated in its present form. It might be that the 24 hour division of the day originated in Egypt—found its way into Babylon and from there into the Hellenistic civilization. The point is not clearly made and because it is so radical the author owes his public a fuller explanation.

In his notes and references at the end of Chapter V the author states, "The myth of the Saros is often used as an explanation of the alleged prediction by Thales of the solar eclipse of—584 May 28. . . . One can safely say that the story about Thales's predicting a solar eclipse is no more reliable than the other story about his predicting the fall of meteors." This passage is so radical that it merits a complete treatment for insertion in an astronomical publication.

This book is bound to have a tremendous impact on astronomical lore and is destined to initiate wide spread revisions in the historical data which have been taken for granted for several generations.

I. M. LEVITT

THEORY OF VIBRATIONS FOR ENGINEERS, by E. B. Cole. 334 pages, illustrations, 14×22 cm. London, Crosby Lockwood & Son, 1950. Price, $3.00.

Recognizing a need for an intermediate text on the theory of vibrations to provide the basis for a better understanding of the more complicated treatises on vibrations, E. B. Cole, lecturer in Mechanical Engineering at the University of Liverpool, has produced a very practical book on the subject. Writing primarily for engineers with only a background in the calculus, Mr. Cole has avoided as much advanced mathematics as possible, while at the same time very ably describing the common engineering problems in vibrations. With a long background in teaching, Mr. Cole wisely explains the theory by interpreting it physically as well as mathematically. Such a method never fails to be successful.

A discussion of simple harmonic motion introduces the theory of vibrations accompanied by discussions of the natural vibrations of systems with one and two degrees of freedom. Damped vibrations, forced vibrations of simple and multiple load systems and natural vibrations of heavy shafts are next treated, followed by some practical applications of the foregoing theory. Separate discussions of engine vibration, the elimination and absorption of vibration, dynamic loads and waves, the traverse vibration of beams and the whirling of shafts conclude the rather thorough coverage of vibrations pertaining to machine design. Problems are included.

As stated above, the book is for engineers, and frequent practical engineering examples reflect this purpose throughout. The brief survey of subjects contained in the book already indicates the engineering nature of the volume. Other examples include the vibration of geared drives, torsional vibrations of crankshafts, earthquake and other vibration recorders and the theory of the friction damper.

E. W. HAMMER, JR.

POLARIZED LIGHT IN METALLOGRAPHY, edited by G. K. T. Conn and F. J. Bradshaw. 130 pages, diagrams, plates, 15×22 cm. New York, Academic Press, Inc.; London, Butterworths Scientific Publications, 1952. Price, $3.80.

This carefully edited little volume contains the results of a critical survey of literature on uses of polarized light in metallographic microscopy, prepared by eight recognized British experts co-opted (sic) by the Optical Methods Sub-committee of the Metallurgy Division of the British Iron and Steel Research Association.

Despite its formidable hierarchic lineage (whose grandeur could, perhaps, be measured fittingly in terms of statute miles of red tape per lineal inch of text), the issue is a well-integrated, well-illustrated and highly informative booklet of 130 pages on an as yet little used optical technique.

Two introductory chapters (40 pages) give a didactically excellent exposition of the concepts of polarization of light and anisotropy of crystalline solids and of the principles of reflection and absorption of light by intervening matter. A third chapter (30 pages) describes the equipment and procedure for microscopy by reflected polarized light.

Addressed primarily to the practicing metallurgist, the three principal chapters (45 pages) present in a well-organized and readable form the technical details, possibilities and problems associated with the microscopic examination of metal surfaces, of inclusions in metals and alloys, and of ores. With the aid of polarized light, the salient features are exemplified by numerous black-and-white photomicrographs.

A list of bibliographic references to original work in the considered fields of application is attached to each chapter.

The concluding chapter (5 pages) by B. W. Mott, gives a practical evaluation of the new technique on the basis of a comparison of its results and potentialities with those of the conventional optical method of observation. A brief glossary of optical terms and a fairly extensive subject index enhance the practical usefulness of this booklet which will serve well as standard reference to the practicing metallographist who ventures to use the new technique.

This reviewer is much impressed by the high quality of presentation at engineering level, maintained throughout the book. He wishes to bring it to the attention of the non-technical reader as an example of good technical writing which not only informs, but also educates the reader to think in accordance with the high standards set by scientific tradition.

OTTO R. SPIES

THE PRINCIPLES OF THE CONTROL AND STABILITY OF AIRCRAFT, by W. J. Duncan. 384 pages, illustrations, 14×22 cm. New York, Cambridge University Press, 1952. Price, $8.00.

This is the first volume to be published in the Cambridge Aeronautical Series. It is intended that this will be a series of aeronautical textbooks on a postgraduate level. The present volume fills the need for a fairly complete, up-to-date text on stability and control of aircraft, although the related theory of flutter has not been treated herein. The mathematical level of the treatment is such that a knowledge of calculus, including some knowledge of linear differential equations with constant coefficients should enable the reader to follow all the mathematical arguments.

After an introductory survey of the subject there are two chapters on the elementary mechanics of flight and the equations of motion of rigid aircraft. The treatment is simplified by omitting degrees of freedom wherever possible so as to simplify while still bringing out the essential details.

The next subject treated is the methods for solving the dynamical equations and for investigating stability. The dynamical systems considered are the linear systems with constant coefficients. The method of "impulsive admittances" and the Routh-Hurwitz stability criterion are discussed.

Longitudinal-symmetric and lateral-antisymmetric motion are then considered. The notion of aerodynamic derivatives for these motions is discussed. Hence a chapter is included on an explanation of how these derivatives can be determined experimentally.

There is a chapter on flap controls in general and a chapter (by A. D. Young) on special flaps for landing and take-off. Another chapter considers the controls which give rise to rolling, pitching, and yawing moments. Stalling and spin are treated in a chapter by Professor Young.

The scope of the book is extended in two chapters which deal with the effects of distortion of the aircraft structure, and the influence of the compressibility of the air as applied to problems of aircraft dynamics.

HAROLD PLATZER

NUMERICAL METHODS IN ENGINEERING, by Mario G. Salvadori and Melvin L. Baron. 258 pages, diagrams and tables, 15×22 cm. New York, Prentice-Hall, Inc., 1952. Price, $5.00.

Those who are familiar with Professor Salvadori's previous book "The Mathematical Solution of Engineering Problems," will welcome this latest addition by the same author. Like its predecessor, this book has evolved from lecture notes used by Prof. Salvadori in his course at Columbia University and is aimed at bridging the gap between a knowledge of theoretical mathematics and the techniques of solving physical problems by mathematical methods.

The book will prove useful to students of engineering, physics, chemistry, mathematics and to any individual desiring to become acquainted with numerical methods in order to apply them in his professional work. Physical problems, used to introduce the mathematical techniques and corresponding theory are taken from various fields of engineering. The solutions are carried through to their final numerical results, a thorough analysis of computational errors accompanying each development.

The five chapters of the book deal with the solution of algebraic equations of high degree and of simultaneous linear algebraic equations; the theory of finite differences and its application to numerical differentiation, integration, interpolation, and extrapolation; the solution of ordinary initial value problems; the solution of ordinary boundary and characteristic value problems; and the solution of problems involving partial differential equations of the boundary, characteristic, and mixed types.

The unifying basis of all the numerical techniques is finite difference theory. This theory is developed in Chapter 2 and then applied to specific problems throughout the remainder of the text. Over 400 problems consisting of both purely numerical exercises and applied examples have been assembled by Melvin L. Baron, a research associate at Columbia.

The text deals with problems more advanced than those presented in the earlier book, but it is clear and concise and should present no difficulty to those having some background in applied mathematics. Coming at a time when numerical methods are increasing greatly in importance, this text should prove to be very popular.

D. L. BIRX

FRACTIONAL H.-P. ELECTRIC MOTORS, by F. G. Spreadbury. 342 pages, diagrams, illustrations, 14×22 cm. New York, Pitman Publishing Corp., 1952. Price, $7.50.

Offering a comprehensive presentation of subject matter, this book deals solely with fractional horsepower motors. The manner in which it is written permits the student and designer alike to use it as a reference and guide. It does not appear to have been written for use as a textbook in a college course of study. Rather, it strikes one as a dissertation of the practical aspects, highlighted with theory, of this type of motor. The mathematics involved in the theory has not been neglected and the potential user will find a working knowledge of differential and integral calculus to be of aid. Enough qualitative material has been included to provide the mathematically uninitiated reader enough "food for thought."

The scope of the book embraces the direct-current motor, single-phase and polyphase induction motors, universal, repulsion, synchronous, and miscellaneous types of motors. In general, each type of motor has been treated in an individual chapter. Other chapters dealing with the construction and design of d-c. motors, performance characteristics of fractional horsepower motors, and motor control make up the ten chapters in the book. An abundance of graphs, charts, oscillograms, and diagrams supplements the subject matter.

It should be noted that the publisher has printed the book on durable paper with a matte surface which diffuses incident light. However, printing the title on the binding so that it must be read with the book in a horizontal position makes it rather inconvenient to store the book vertically.

E. A. FASKO

ACROSS THE SPACE FRONTIER, edited by Cornelius Ryan. 147 pages, illustrations, plates, 21×28 cm. New York, The Viking Press, 1952. Price, $3.95.

Within the past two years there have appeared several books and other publications on what may well be the supreme achievement of man—the conquest and exploration of space.

The popular books by Willy Ley, by Chesley Bonestell and Willy Ley, and by Arthur C. Clarke have whetted the appetites of the space travel enthusiasts. Now within a month two more books have appeared—"Physics and Medicine of the Upper Atmosphere" and "Across the Space Frontier."

Both of these are the results of symposia; both of them are well written and carefully edited and both are musts in the library. One is heavily scientific and 611 pages long, whereas the other is a skillfully written book on a popular level.

"Across the Space Frontier" is an outstanding description of the space platform of the future—the starting place for interplanetary travel. The contributors are the ranking scientists in their respective fields.

Dr. Joseph Kaplan is Professor of Physics, at the University of California, and he details our knowledge of the atmosphere.

Dr. Wernher von Braun is with the Army Ordnance Guided Missile Development group and is one of the designers of the V-2. He discusses the development, mechanics, and operation of the space station.

Dr. Heinz Haber, assistant Professor of Astrophysics, University of California, describes the behavior of the human being in space.

Willy Ley, the popularizer of space travel, par excellence, is at his best in discussing the everyday events transpiring in a space station.

The legal problems of ownership and motion of the space ship are excellently covered by Oscar Schachter.

Dr. Fred L. Whipple, Chairman, Department of Astronomy, Harvard University, discusses the many problems which will find their solutions with the operation of a space station astronomical observatory.

To Cornelius Ryan, Associate Editor of *Collier's*, must go the lion's share of credit for the outstanding book on space travel. As editor of the book his excellent judgment prevailed in selecting the topics and having them presented in a clear lucid manner. Apparent throughout the book is the clear incisive writing characteristic of the "slick paper" magazines and the fine hand of Mr. Ryan and his associates is apparent throughout.

This book contains remarkably fine illustrations by the most skillful of the technical artists. The inimitable Chesley Bonestell has been aided by Rolf Kelp and Fred Freeman and their combined efforts are effective in presenting the visual accompaniment to the text.

Seldom has the layman interested in science been so indebted to a popular magazine, for without the sponsorship of *Collier's*, "Across the Space Frontier" would have been an impossibility.

I. M. LEVITT

METALLURGY FOR ENGINEERS, by John Wulff, Howard F. Taylor and Amos J. Shaler. 624 pages, diagrams, 15×23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.75.

Written in a clear, sometimes excessively simple style, "Metallurgy for Engineers" is another contribution to scientific literature noteworthy primarily because it conforms to a rather recent trend in technical books in which a subject is delineated so that the novice may easily understand the essential concepts. One studying metallurgy as a major subject would probably not bother with this book, since it is elementary. But for one studying engineering and interested in acquiring an over-all comprehension of an important field of engineering, "Metallurgy for Engineers" is just the thing. It provides a good elementary introduction to metallurgy.

Compiled by three staff members of the Metal Processing Laboratory of Massachusetts Institute of Technology, the book reflects the experienced teacher. Concepts are stated simply, and at the end of each chapter are summarized even more simply. Questions further emphasize the significant facts.

The book is divided into basic metallurgy and casting processes. Beginning with a very elementary introduction to engineering metals, the authors discuss such basic principles of metallurgy as structure and properties of metals, impurity and solubility, phase equilibrium, alloy systems, ferrous and non-ferrous alloys, heat treatment, and measurement of mechanical properties of metals and deformation of metals. In the second part the authors discuss various details of casting with the emphasis on sand casting; powder metallurgy; such various welding processes as arc, gas, thermit and pressure; brazing and soldering; and metal working.

E. W. HAMMER, JR.

BOOK NOTES

DICTIONARY OF CONFORMAL REPRESENTATIONS, by H. Kober. 224 pages, diagrams, 16×24 cm. New York, Dover Publications, Inc., 1952. Price, $3.95 (cloth).

Developed for the British Admiralty, this unusual book contains scores of geometrical forms and their transformations for the use of engineers, aerodynamicists, and physicists, in checking solutions to problems. Classified according to the analytic functions describing the transformations, the "Dictionary" is easily used to find time-saving set-ups. Among the specialized features are: linear and bilinear transformations for electrical engineers; Joukowski aerofoil for aerodynamicists; transcendental functions for physicists and engineers; Schwartz-Christophel transformations for hydrodynamicists and electrical engineers; Borda's mouthpiece for researchers in hydraulics and jet propulsion. Most transformations are illustrated by pairs of diagrams which show corresponding curves and corresponding regions.

HIGHWAY CURVES, by Howard Chapin Ives. Fourth edition revised by Philip Kissam. 389 pages, diagrams, plates, 11×18 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1952. Price, $7.00.

This completely rewritten fourth edition of a standard handbook was necessitated by the spectacular advances in highway design since the appearance of the previous edition in 1941. Designed to simplify both the field operations and the mathematical solutions, the present edition emphasizes simplicity of computation and flexibility of field methods. New tables are included which eliminate the need to use logarithms or a calculating machine to compute values for a simple curve. For compound curves the Searles spiral has been selected as the best, and complete tables for its use are included. Chapters on cross sections and interchanges should prove useful.

ELECTRICAL MEASUREMENTS MANUAL, by C. H. Dunn and H. J. Parker. 112 pages, diagrams, 15×22 cm. New York, Prentice-Hall, Inc., 1952. Price, $3.25.

Written as a text for an elementary laboratory course in electrical measurements, this little manual gives an extensive treatment of laboratory techniques in addition to a well-balanced selection of laboratory experiments, on the characteristics of basic electrical instruments and the use of these instruments in measuring electrical and magnetic quantities. Many of the experiments combine the study of measurements technique and circuit theory, and so the manual can be adapted to circuit theory laboratories as well as measurements laboratories. The experiments are preceded by a brief discussion and a listing of the necessary equipment. The procedure is given and a section labeled "Conclusions" contains several questions designed to point to some of the important conclusions to be drawn from the experiment. This should prove to be a useful text.

PHYSICS FOR MODEN TIMES, by A. Marcus. 762 pages, diagrams, 15×22 cm. New York, Prentice-Hall, Inc., 1952. Price, $3.92.

Written by a high school teacher for high school physics students, this new text is intended for a general physics course, not a college preparatory one. The author approaches physics as a cultural, not a technical, subject. As a result, we find sections on the vacuum coffee maker, the pressure cooker, the refrigerator, the washing machine, the automobile, musical instruments, radar, television—along with the more usual subjects. In his attempt to give the student an understanding of the objects surrounding him, the author has simplified the study of physics to such an extent that the text contains only a few formulas and no numerical problems. The only questions are discussion questions. While this approach may be an interesting one for the non-scientific student, this reviewer is of the opinion that too much ground has been covered at too great a speed, and at a sacrifice of the main tenets of science—objectivity and rigorous proof.

OBSERVER'S HANDBOOK. 216 pages, illustrations, 16×20 cm. London, Her Majesty's Stationery Office, 1952. Price, 12s 6d.

This is the authoritative manual of surface meteorological observing. It covers all standard observations at all types of meteorological stations, whether synoptic stations or climatological stations. Various types of highly specialized observations, including upper air observations, do not come within its scope.

The new "Handbook" differs from its predecessors in two important respects. It has been written with the needs of modern meteorology in mind, and due prominence is therefore given, in the proper sections, to the international codes which have been in use since January 1, 1949. Secondly, the body of the book is concerned with observing proper and it is therefore assumed that the station has been properly cited, equipped on the scale appropriate to its class and is in full working order. The first chapter contains a summary of the observing procedure at each type of station, with some general notes on observing, and the following chapters are devoted in turn to the details of observing the separate elements.

Appendices cover the selection of the site and other matters which must receive attention when the station is first set up, the recording of observations, and the preparation of barometer correction cards.

PHOSPHORIC ACID, PHOSPHATES AND PHOSPHATIC FERTILIZERS, by William H. Waggaman. Second edition, 683 pages, illustrations, diagrams, 16×23 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

After 25 years, the ACS monograph has been revised to conform to a more recent policy of confining a monograph to a complete and critical treatment of a relatively restricted area. This second edition is almost completely new, and, because it contains the latest information treated at length, should become the leading source of information on all phases of phosphorus. To be especially noted is a unique treatment of the economic aspects of manufacturing, both from the manufacturer's and the consumer's viewpoints. A complete list of U. S. Patents

on the manufacture and use of phosphorus compounds will be of great value to research workers. The monograph would be of interest to technical men engaged in the recovery and processing of phosphate rock, to plant superintendents, to research chemists, and to those in industries which use phosphorus in some phase of their operations.

OXIDATION POTENTIALS, by Wendell M. Latimer. Second edition, 392 pages, 15×23 cm. New York, Prentice-Hall, Inc., 1952. Price, $7.50.

In this second edition, the author has extended the coverage of thermodynamic data which are of special significance in the interpretation of inorganic chemistry. Potential diagrams are now given for all the elements which have more than one oxidation state, and several hundred new potential values and equilibrium constants have been added. The discussions of uranium, neptunium, plutonium, and americum have been greatly enlarged and the actinide elements are treated in a new chapter. A set of study questions for use in conjunction with a course in advanced inorganic chemistry has been included in the appendix. This second edition should be considerably more valuable than the first, since it has been brought up-to-date.

THE SCIENCE OF FLAMES AND FURNACES, by M. W. Thring. 416 pages, illustrations, 14×22 cm. New York, John Wiley & Sons, Inc., 1952. Price, $6.50.

The author has prepared this monograph on industrial furnaces in an attempt to focus attention on the gaps in our scientific knowledge of furnace design and operation, and to make more accessible to designers and operators what knowledge there is at present. Feeling that the furnace itself has been neglected while the allied fields of the internal-combustion engine, convection heating and turbulence and fluid flow in aircraft design have been thoroughly pursued, the author hopes through his book to interest more research workers in the fundamental problems of the furnace. Some of the problems to be solved or studied are: pre-heat, use of waste heat, recirculation of flue gases, quality of fuel. The book is limited to furnaces heated by flames. References in the form of footnotes are provided for those who wish to follow up the subject.

THE FRANKLIN INSTITUTE

exists today because of the faith and generosity of the men and women who for 128 years have given time and money to its support.

The Institute welcomes financial gifts and bequests and hopes that all those who desire to perpetuate its work will make the The Institute one of their beneficiaries.

FORM OF BEQUEST

*I give and bequeath to The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts, the sum of*_____

_____

CURRENT TOPICS

Proposed Electrical Code Changes.—Proposed changes for the 1953 National Electrical Code have been released in Boston by the National Fire Protection Association, sponsors of the Code. Published in NFPA Pamphlet No. 70-PRI, the changes, proposals not recommended and items remaining on the dockets of the various "Code Making Panels" are detailed and circulated for public comment. A deadline date of December 1, 1952 has been established for all comments if they are to receive Committee consideration prior to the final drafting of the 1953 Code.

The pamphlet, available for $2.00 per copy, illustrates the democratic method followed by the NFPA in preparing its standards. Industry, inspection authorities, fire officials, governmental agencies, insurance groups and cooperating national Associations participate in developing the Code and the general public is afforded this opportunity to comment.

Some of the significant proposed changes and additions follow. Electrical receptacles will henceforth be required to be designed so that they cannot be used interchangeably in different circuits of different voltage, frequency or types of current (a.c. or d.c.). New rules are proposed to safeguard users of portable equipment and electric clothes driers by more effective grounding means. Liberalization of the use of armored cable in the air voids of cinder block walls is suggested except where subject to excessive moisture or dampness.

A revised set of markings for flexible cords and fixture wire is prepared to eliminate obsolete and include new types, affecting particularly heat-resistant, rubber-covered fixture wire, latex-rubber insulated wires, tinsel cords, heater cords, and range cables. Provisions are proposed to prevent the installation of fixtures and outlet boxes in contact with conducting surfaces of conductive thermal insulation. Gas piping is to be deleted as a method of grounding electrical circuits.

The NFPA Code Making Panel dealing with electrical appliances recommends that it be made mandatory for flatirons and water heaters to be equipped with temperature limiting devices, which action will undoubtedly go far to eliminate fires caused by accidental overheating of such appliances.

A special subcommittee is submitting to the NFPA and the public a proposed new section on fixed electrical space heating equipment in recognition of the growing use of such means of home heating. Another appliance receiving attention is refrigerator motors which the 1953 Code will require to have adequate ventilation to prevent overheating by a change in Section 4310 of the 1951 Code.

Significant changes are proposed for the section of the Code dealing with "hazardous locations," particularly Articles 500 and 510. Recognition has been given to positive pressure ventilation arrangements to reduce the area considered hazardous when such ventilation is safeguarded by duplication of equipment or interlocked with the electrical supply. Specific temperature limitations are suggested for surface temperatures of electrical equipment in atmospheres involving Class II or III dust explosion hazards, that is atmospheres laden with combustible dusts, fibers or flyings.

Perhaps the most sweeping changes of all are in Article 510 where complete new rules are proposed for commercial garages, residential garages, aircraft hangars, gasoline service stations, bulk plants storing flammable liquids, paint, lacquer and similar finishing process areas, and where combustible anesthetics are used.

Other complete revisions affect Motion Picture Studios, Article 530, and elevators, Article 620. The latter article also is broadened to take care of dumbwaiters and moving stairways (escalators). A completely new Article 680 is proposed to cover hermetically sealed refrigerating machines.

Emergency lighting systems, as are generally required for hotels, theaters, sports arenas, hospitals and similar institutions also receive a complete new treatment. Requirements for emergency power, as needed for essential refrigeration, breathing apparatus, power for hospital operating rooms, fire pumps and industrial processes where current interruption would have serious consequences, are added to the previous rules which covered only lighting. The new Code proposals feature *how* emergency systems should be installed leaving to other NFPA Standards, specifications as to *where* and *when* such systems are needed. The NFPA Building Exit Code is the principal standard defining the latter requirements.

A general revision of Article 710, applying to circuits and equipment operating at more than 600 volts between conductors, is also proposed, this particular section needing clarification to indicate clearly its intent. Article 725 is broadened to include low voltage circuits not used for signal or remote control circuits to which previously this Article was restricted. Article 810 has had its title changed to clearly indicate its application to television as well as radio equipment and important revisions are made to more specifically help installers on the installation of antennas and to detail methods of grounding.

Copies of the pamphlet are available by addressing NFPA Publication Services, 60 Batterymarch Street, Boston 10, Mass. The cost is $2.00 per copy, necessitated by the high printing expense of the 226 page pamphlet by the non-profit technical Association.

Newly Designed Three-Piece Gear Motor.—A new gear motor, designed to simplify maintenance in case of electrical failure of the stator, has been announced by the General Electric Company's Small and Medium Motor Department.

The new motor is built in three pieces to permit quick and easy removal of defective stators without disturbing gear components or gear connections to the load. Its unique design includes a G-E Tri-Clad* motor; a compact, planetary gear; and a mechanical adapter.

G-E Tri-Clad motor construction offers extra protection against operating wear and tear, physical damage, and electrical breakdown.

The gear, developed last year by G-E engineers, is available in speed ratings of 780 to 13.5 rpm.

The mechanical adapter serves the triple function of acting as the pulley end-shield for the motor, the motor end-shield of the gear housing, and a mechanical spacer for easy accessibility to the stator frame bolts.

According to G-E engineers who worked on its development, the new

* Reg. Trade Mark of the General Electric Co.

design will provide maximum continuity of service with minimum spare parts inventory. It is expected to cut stator replacement time in half, they said.

The stator can be changed with the removal of only bearing-enclosure screws, the bolts that hold the front end-shield to the stator, and bolts holding stator to the motor adapter. A defective stator can be replaced by either another round-frame stator or by an ordinary, standard-motor stator with feet. If a standard stator is used, it is necessary to mount it upside down so its feet do not interfere with the gear-motor feet.

In the new three-piece construction, openings for motor ventilation are provided at the bottom. This means that all G-E open gear motors are now dripproof.

The new motor is available in ratings of 1 to 75 hp. for standard applications and up to 200 hp. for special applications.

New Type Data Recorder and Annunciator.—Taller & Cooper, Inc., of Brooklyn, has announced a new sequential automatic data recorder and annunciator for use in industrial plants and power systems. In addition to providing a complete annunciation system to warn of danger and forestall damage, it creates a running log on the condition and on the measures taken to maintain satisfactory operation. Since the Recorder-Annunciator provides a sequential record, engineers have an instantaneous analysis of any breakdown, which enables them to go directly to the basic failure and rectify it. The ability of the system to pinpoint the origin of any stoppage saves hours of searching and the curtailment of production is reduced to a minimum. An accurate record of sequential equipment operation during failure may save great costs in alterations and in the installation of new equipment.

The Recorder-Annunciator consists of four main units: the fact-collecting unit, the read-out unit, the printing recorder unit and the annunciator unit. Wires from hundreds of trouble contacts, mounted on the equipment throughout the plant, lead to the coding relays in the fact-collecting unit. Thus information is gathered on the operating condition of the plant. The coding relays transpose the information into electrical patterns (coding) and transmit these to the electric memory where they are stored in the order of arrival, one fact after another. The facts now retained in the electric memory are picked out in order by the sequential memory read-out, which sends the facts, still in correct order but at a rate that the printing recorder can handle, to the decoding section. The decoding section translates the electrical patterns into words by setting up the correct designations on the print wheels in the printing recorder. As each "fact" comes through, the print wheels spin to the appropriate position and a recording is made. The paper advances, a new "fact" is set up and the printing recorder prints again, fashioning in this manner a permanent record on a continuous roll of paper. The annunciator, which gives visible and audible warning whenever trouble arises, is connected to the coding relays in the fact-collecting unit. Operating simultaneously with the receipt of impulses to the relays, it need not await the memorizing, decoding and printing cycle of the recorder.

The custom applications of the Sequential Automatic Data Recorder and Annunciator are unlimited. In general, it can be used in any plant where many simultaneous operations must be continuously recorded or checked.

In this sense it is a great eye constantly watching as many as five hundred separate operations. It is excellently suited for airport, bus and rail dispatching, as well as for industrial plants such as oil refineries, steel mills or paint factories where production and quality control are essential.

Fireproof Plastic for Interior Decorating.—A fireproof, decorative Micarta plastic was perfected by the Micarta Division of the Westinghouse Electric Corporation to satisfy the luxurious yet safe requirements for the interior furnishings of the superliner *S. S. United States*. This is the same vessel—powered by Westinghouse geared steam turbines—that recently broke all trans-Atlantic speed records. This new type of Micarta is in use on hundreds of the vessel's table tops, chairs, and other furniture surfaces.

"After months of experimentation, engineers at the Micarta Division came up with this amazingly tough, fire-repellent material," Mr. E. R. Perry, division manager, said. "We believe it may be the long-awaited answer to many decorating problems in office and other public buildings as well as in homes, where safety is a major factor in construction and decoration."

The most exacting of decorators' needs can be met with this new material since it can be made in a wide variety of colors and wood-grain or textile finishes. To meet the needs of the superliner's decorators, 14 different colors were furnished in both satin and glossy finishes, ranging from plain white to beige, grey, shades of green and red, and black.

The new material is formed in a single process by sandwiching a $\frac{1}{16}$-in. aluminum sheet between layers of melamine plastic. Since aluminum and melamine plastic expand at different rates for a given change in temperature, a way had to be found to form a firm and lasting bond between these materials. This problem was solved by attaching them with a layer of phenolic resin about equal in thickness with the decorative melamine layer. This plastic-to-metal bond is made simultaneously with the pressure forming of the outer decorative melamine layer. The bond developed is so strong that it can be subjected to widely fluctuating extremes of temperature as well as humidities up to 100 per cent, without breaking.

Mr. Perry explained that safety of the new plastic material from a fire-repellent standpoint comes from the heat-conducting aluminum and the melamine coating. The foundation aluminum sheet disperses heat that may be caused by such things as hot food containers or cigarettes, thereby virtually eliminating the danger of fire. The melamine outer layer does not sustain combustion, though it will char when heat is applied for an appreciable length of time.

New Electronic Brain.—A new and improved electronic "brain" capable of working the most difficult and complex mathematical problems has been developed by Goodyear Aircraft Corporation, Akron, Ohio. The L3, amazing new version of GEDA (Goodyear Electronic Differential Analyzer) can solve in one hour scientific problems that would require as much as 500 hours of advanced computation by engineering specialists.

Designed and developed by Goodyear Aircraft, the analyzer has been used mostly for classified work under government contract. The new L3 GEDA was also built by the company as a commercial product to permit its use in a variety of fields.

Usefulness of the machine is evident not only to electronic experts but to the industrialist as well. In a general sense, the analyzer is an outsized slide rule and as such, constitutes an answer to the critical shortage of engineers—in all fields. With its ability to determine the solution to problems of higher mathematics in a matter of minutes, the GEDA permits engineering specialists to devote more time to advanced phases of their research and development activities.

The newest member of the family of Goodyear computing equipment is the result of five years of development activity. It is capable of solving problems of motion and control either by direct simulation or by mathematical equations covering a multitude of fields ranging from questions dealing with flight in solar space to the measure of a baseball's curve.

The analyzer solves problems in terms of voltages and wave forms, which are transcribed onto paper in allied recording machines, and interpreted by trained technicians. Problems of direct simulation are answered through a study of electrical and mechanical circuits. By substituting different voltages in the L3 GEDA for the different parts of an equation, any problem of motion that can be translated into a mathematical equation can be solved.

Use of the Goodyear analyzer encourages advanced thinking and inventive processes through its ability to test the workability of new ideas. Design evaluation and accurate study of product reliability are other important uses of the L3 GEDA.

Smaller and more compact than the usual computer unit, the L3 gives the appearance of being an ultra-modern, streamlined telephone switchboard. It is operated entirely from the front, either from a sitting or standing position. The control panel is sloped for easy reading of dials and indicators.

A removable problem board, which directly faces the operator, is made of light glass-fibre, reinforced plastic. It is easily removed for wiring purposes. Space is also provided between the control panel and the problem board to permit the mounting of various auxiliary equipment.

Maintenance is simplified by use of plug-in units and by the design of panels that swing out, up, or down, making almost all parts accessible without unsoldering or removing a single wire.

Through use of the computer, a great deal of the guesswork in aviation is being eliminated. In addition to a study of everyday flight problems, the GEDA is used widely in the study of airplane and airship flight and in the analysis of pilotless aircraft systems, including guided missiles. Many navigation problems of tomorrow, such as a possible flight to the moon, or formation of a satellite body outside the earth's atmosphere stand within the reach of engineers with the aid of Goodyear Aircraft's mechanical "brain."

Designed to simplify engineering problems, work for the L3 GEDA can be found almost anywhere in industry. Uses for the machine have already been found in the textile and automotive fields. Evident is its ability to find the answer to complex engineering problems in the fields of heavy construction, mining, petroleum recovery, transportation and in the field of atomic research and development. The technological skill of engineering students is also being enhanced through the use of GEDAs.

New Type Paint Remover.—A new type electric paint remover, designed to remove paint quickly and economically through the use of radiant heat, has

been anounced by the Industrial Heating Department of the General Electric Company.

Developed for both professional and home use, the new paint remover consists of two skids under which is mounted a G-E Calrod* tubular heater rated at 1000 watts, 115 volts. When the device is slid along a painted surface, the heat from the Calrod unit causes the paint to soften and blister and it can then be removed with a putty knife.

Old paint can be removed continuously as fast as the operator can scrape it off, according to G-E engineers. The skids keep the heat a safe and constant distance from the painted surface while two metal reflectors keep the plastic handle cool at all times.

The complete device, to be sold through Sherwin-Williams paint stores, weighs 2¼ lb., is 6½ in. long, and includes an eight-foot heavy-duty electric cord.

Reactor-Produced Polonium Now Available for Purchase.—The Isotopes Division of the Atomic Energy Commission has announced that reactor-produced radioactive polonium-210 may now be purchased at Oak Ridge for research activities. Polonium is the first reactor-produced radioisotope to be sold which emits alpha particles. It can be used also as a source of high energy neutrons.

The Isotopes Division, a part of the AEC's Oak Ridge Operations, and the U. S. licensing agency for purchases of significant quantities of radioactive materials, said the newly available polonium is needed by research groups for physical and biological investigations. It may be used also in oil well logging and for ionization sources.

In addition, the element could be used in luminous phosphors, static elimination devices, and for other industrial purposes. However, it is not available at present in the quantities required for such uses.

Polonium can be bought at Oak Ridge National Laboratory in two forms. When it is to be used as a neutron source, the polonium will be mixed with beryllium and enclosed in a cylinder of nickel, about ¾-in. in diameter. As an alpha source, the polonium will be plated on a strip of platinum.

Polonium-210 produced in a reactor by neutron bombardment of bismuth is of higher purity than polonium derived from the radioactive decay of radium.

Rainfall Patterns.—Investigation of recent rainfall patterns recorded throughout much of this country has produced new evidence of close correlation between periodic cloud seeding and periodic weather changes, according to Dr. Irving Langmuir, noted General Electric Company scientist.

Speaking at a meeting of the Institute of Mathematical Sciences, he said that periodic commercial seedings last spring in western states during the middle of the week "correlated extremely well" with rainfall patterns occurring later in the week in eastern states.

"These periodicities showed up generally from coast to coast," he said. He pointed out that the phases of rainfall moved in orderly progression across the entire country.

Dr. Langmuir previously has reported that periodic seedings he supervised in 1950 in New Mexico showed high correlation with regularly-spaced rainfall.

He said that agreement between rainfall and seeding phases in that project

* Reg. Trade Mark.

"gave conclusive proof that the periodicities were actually caused by silver iodide seeding." He pointed out that these periodicities showed up when weather data from the entire country was studied, but that "the country never is uniformly affected. One state may be affected more than another."

The famed G-E Research Laboratory consultant said today that first reports of the periodic nature of the weather patterns last March and May seemed to refute earlier findings of high correlation between seeding with silver iodide crystals in New Mexico. This was because there had been no reports of periodic seeding last spring, he explained.

But a thorough study of commercial seeding operations in the West last spring proved that these seedings had been conducted periodically or at regular intervals, according to Dr. Langmuir.

He said that data supplied to the G-E weather scientists in Schenectady, N. Y., by seeding firms carrying on large scale operations in western states showed that seeding was predominant in the middle of the week during March and May, 1952.

Regular mid-week seeding took place in the Far West last March. Similar operations were carried out in March and May this year by seeders in some midwestern states.

It is possible, also, that other commercial seeders farther east took advantage of favorable clouds produced by these seedings and further intensified spaced rainfall as the weather moved east toward the Atlantic Coast, Dr. Langmuir stated.

The number of periodic seedings that took place during March and May in the West, he pointed out, "was many times more than we conducted in 1950." The periodicity of weather changes was greater during last March and May, he said, even though "the effects were not as clean-cut as they were in April, 1950."

Dr. Langmuir also contrasted his recent findings with those reported in the May issue of the "Monthly Weather Review," published by the U. S. Department of Commerce Weather Bureau. The article by F. H. Hawkins, Jr., stated that as far as could be determined, no seeding was under way last spring which could compare in periodicity with the marked spacing of rainfall at that time.

Despite this, however, Weather Bureau data gathered from the eastern states showed "a preference for precipitation to occur on Sunday with decreasing amounts on either side and no measurable precipitation on Wednesday and Thursday."

Dr. Langmuir said that extensive analysis of seeding dates supplied by seeding firms showed that artificial influencing of rainfall occurred in definite patterns throughout the West. These data disclosed that periodic seeding "was at least 20 times heavier in May, 1952, than any conducted before," Dr. Langmuir commented.

The meeting was co-sponsored by the American Meteorological Society and the American Geophysical Union.

World's Most Powerful Jet Engine.—The Navy and the Westinghouse Electric Corporation have announced that Westinghouse has developed and placed in production the world's most powerful qualified turbojet aircraft

engine. It develops thrust equivalent to approximately 25,000 horsepower at today's jet flight speeds.

"The new power plant will be the first jet engine in the world to provide constant speed drive for airplane accessories as an integral part of the engine," said Frank L. Snyder, manager of the Westinghouse Aviation Gas Turbine Division, Philadelphia, Pa. "This revolutionary feature will permit designers to make substantial savings in weight and space in new planes, both vital factors in maintaining aircraft superiority."

The new aircraft power plant has just successfully completed the Defense Department's grueling 150-hr. qualification test, which all new engines must pass to be eligible for quantity production. The test was equivalent to more than 75,000 miles of actual flight, or three times around the world at the equator.

Several new fighter planes, all of which are scheduled for early production for the U. S. Navy, are expected to be powered by the new engine. These will include the McDonnell F3H "Demon" and the Douglas F4D "Skyray."

The new engine is an outgrowth of the original Westinghouse J40 engine which, when announced last year, was the most powerful turbojet in the world to qualify for production. The new engine thus assumes the role of leadership previously held by the earlier model.

The tremendous power of the new J40 is partly developed through the use of an after burner which reheats the exhaust gases after they leave the turbine but before they emerge as a jet stream, Mr. Snyder revealed.

The new engine, like all Westinghouse jets, is of the axial flow or straight-through design. Almost 25 feet long and about 40 inches in diameter, it is exceptionally light and produces more thrust per square inch of frontal area than any other turbojet.

"This single engine is two and one-half times as powerful as the combined four engines on a B-29 Superfortress," Mr. Snyder declared, "but its weight, approximately 3500 pounds, is less than that of only one of the famous bomber's engines and propellers. And still more powerful versions of the J40 are under development."

"Westinghouse metallurgists and engineers are constantly searching for ways to reduce the quantities of critical materials that go into jet engines," Mr. Snyder added. "Substantial amount of two of our most critical metals—columbium and cobalt—have been eliminated in the new J40. Even greater savings of these and other scarce materials will be effected in newer versions of the engine."

A new low-cost fuel especially developed for use in high-flying jets and designated as "JP-4," was used throughout the qualification test here, engineers reported. The new J40 thus becomes the first Navy-sponsored turbojet to qualify on the new fuel.

Butt welding of very thin stainless steel is possible through use of a condenser discharge technique perfected by workers at the Naval Ordnance Laboratory, White Oak, Md. Electrical energy stored in a bank of capacitors is discharged through transformers into the weld, in periods ranging from 0.001 to 0.010 second. With this technique, it is possible to weld T and I beams with flanges 0.135 in. wide and 0.0083 in. thick, with web 0.25 in. wide and 0.0077 in. thick.

Fluorescent tube disposal has been made less hazardous at the Pearl Harbor Naval Shipyard with the development of a powered tube breaker fitted with a conical base by which refuse can be funneled into a steel drum. Weights swinging from a fan-motor-driven shaft pulverize the tubes as they are pushed into the breaker, and four sprinkler nozzles settle phosphorus particles and flush them into the drum. About 230 4-foot tubes can be broken into one 55-gallon drum, after which it is sealed and buried. The tube breaker is self-cleaning, eliminating any chance of cuts and scratches.

New steam catapult for aircraft carrier, developed by Great Britain and tested earlier this year by the U. S. Navy, will be manufactured in U. S. by McKiernan-Torry Corp., Harrison, N. J., and E. W. Bliss Co. of Canton, Ohio. First "steam slingshot" will be assembled with some parts shipped from Scotland, some made by these two firms, will be shore-based at Naval Air Material Center, Philadelphia, for test purposes. First shipboard installation will be in *U.S.S. Hancock*, now being modernized at Bremerton Navy Yard.

Increased sensitivity of ground-station receiving equipment used in connection with missile-borne telemetering systems has been made necessary by a trend toward higher altitudes which in turn demanded lighter weight equipment with increased range. The Naval Research Laboratory, ONR, in working on this problem, has attained a noise figure of 2.5 db at 225 mc. with a microwave triode as a grounded-grid amplifier. Two circuits, designed with lumped parameters, showed gains of approximately 10 db and bandwidths of approximately 70 mc.

Mercury Jet Switch (Adapted from "The Mercury Jet Switch and its Applications," by Rodney Hoff, a paper presented at the Seventh Conference of the Instrument Society of America, September 8–12, 1952, in Cleveland.).—The mercury jet switch is a high-speed, electrical, multiple-point selector switch which has very low switching transients. It was developed to afford rapid scanning, monitoring, and displaying of a large number of electrical signals. Instrumentation which it permits should be very useful to many phases of the chemical, petroleum, and aircraft industries.

The mercury jet switch is considerably faster than switches of comparable noise level that were revealed by a search of the literature. A jet switch, for example, has been operated at rates up to 1800 points per second with a noise level of 20 microvolts across 200 ohms.

The mercury jet switch consists of a Bakelite commutator having a circular arrangement of contact pins about a rotor. A fine jet of mercury is emitted from this rotor to scan each contact in sequence as the rotor revolves. Mercury wetted contacts are used so that switching takes place between liquid mercury surfaces. This identity of contact surfaces and the absence of rubbing mechanical friction combine to virtually eliminate switching transients.

The mercury jet switch and its equipment for application to a temperature mapping system have been built and tested. The jet switches have been operated for several thousand hours without any apparent wear or abrasion of the contacts. Hence, it is felt that the feasibility of using a rotating jet of mercury as the selector contact in an electrical switch has been demonstrated. The switch, or various modifications of it, should find many applications wherever rapid scanning of multiple leads is necessary.